Math problems of samurai period

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1 Short introduction

The area of history of mathematics is primarily an investigation into the origin of discoveries in mathematics and, to a lesser extent, an investigation into the mathematical methods and notation of the past.

One possible direction to present math arguments and standard lessons in classroom is based on combination of some elements of the history of mathematics and the classical math arguments during math teaching . There are several practical questions that need to be discussed in order to clarify how this can be done. In other words, what one has to worry about in combining history and mathematics education is chiefly how to do it:

- What examples should one chose for what material?
- What kind of history of mathematics activities can be incorporated into the ordinary mathematics curriculum?
- How does one find time for such activities?
- How does one find a place for history of mathematics in teacher training?

Since our goal in this section is to give some examples and then to try to apply them in a concrete course for future math teachers, we shall not concentrate on the answers to above questions, but shall try to take them into account in the practical realization of the course for future teachers.

It is well known that Renaissance, slowly prepared by Christian and Jewish mathematicians, blossomed first, as we should expect, in Italy, then in the Netherlands, England, and the other countries of Europe, where trade was flourishing and new cities rapidly growing, where universities vied with one another, and emulation was excited by proud challenges from some of the mathematicians to their rivals.

The experience of the group math competitions described in the book [1] shows that one can make some extra curricula math activities very attractive and successful by making reference and recalling some of the traditional facts from the Renaissance period. More precisely, the idea of competing mathematicians was not alien to the Italian history, to mention just the public contests between Tartaglia and Fior, or between Ferrari and Tartaglia in solving algebraic equations. (The problems were presented in front of Notary, then they were printed and distributed in Italy, which stimulated the math research during the Renaissance.) Thus the competition Torneo Vinci was born as a component of The Unsung Hero with the hope to inspire more young people to enrol in math studies. Further details can be found in [1].

We continue this experience modifying the subject a little bit. This possibility occurs, since Prof. Kurokawa was a guest of the Department of Mathematics in the period april - september 2011.

Therefore, we shall see concrete examples how the Japanese history of Mathematics can be implemented in standard and "non so standard" math lessons.

2 Math exmaples from Edo period in Japan

Higher mathematics in pre-modern Japan, also known as wasan, flourished during the Edo period from the early 17th to the late 19th centuries. It had its origins in the Chinese mathematical texts which were acquired by the Japanese during their invasions of Korea in the late 16th century. In a fairly isolated state, the Japanese mathematicians were able to make brilliant new discoveries in areas rarely attributed to a non-Western scholarship, sometimes outpacing their Western counterparts.

The real development of Japanese mathematics began after the invasion of Korea in 1592. A soldier was able to return to the port of Hakata with a Chinese abacus, which existed in China since the 1200s and became known as soroban in Japanese. Its use became widespread after Mori Shigeyoshi published an introductory text regarding the abacus in 1622, although it did not completely replace the sangi that was more convenient for complex algebraic operations. A more extensive text, and also the first complete mathematics book in Japan, was published under the title, Jinkoki, or Large and Small Numbers, in 1627 by Yoshida Mitsuyoshi. This was originally based on Chinese books and used as a textbook in the temple schools at the time. This contains not only basic calculation but also some useful problem for life in Edo period and some mathematical games. One can see [5] for more information.

After this initial takeoff, Japanese mathematics steadily progressed, primarily in areas of geometry and number theory, within an open intellectual dialogue that was initially facilitated and proliferated by the samurai class. At the onset, mathematics was mostly pursued by the samurais as it had military application in surveying, navigation, and calendar making. As the samurais came to work as civil servants with modest stipends, they also taught reading, writing, and arithmetic in small private schools (usually in temples, see Figure 1) called juku, which comprised much of the educational system in pre-modern Japan. Because of the low attendance fees, jukus were widely attended by members across all ages and economic strata, and consequently mathematics became widely accessible within the Japanese society. People who could not afford to publish their own books posted their new findings on wooden prayer tablets in temples as offering to the gods. These tablets, which became known as sangaku, allowed the Japanese mathematicians to exchange ideas and identify new problems and thus transformed the temples into intellectual forums facilitating a nationwide dialogue.

Figure 1: Temples were typical place for samurai classes

Most of the exercises in the Jinkoki are concerned with calculations useful for everyday life and business transactions. We start with typical example.

Exercise 1. *(Thieves Arithmetic) One night, some thieves steal a roll of cloth from a shed. They are dividing up the cloth under a bridge when a passer-by overhears their conversation: If each of us gets 7 tan*[1]*, then 8 tan are left over, but if each of us tries to take 8, then were 7 tan short.2 How many thieves were there, and how long was the cloth?*

Hint. If N is the number of thieves and L is the length of the cloth, then the first condition tells us that $7N = L - 8$. The second condition says $L = 8N - 7$. Solving these two equations gives $N = 15$ and $L = 113$ tan. The problem of the silk thieves appeared in Yoshidas Jinkoki of 1631.

It is important to note that the proposed problem is a real life problem. The solution is not difficult, but the real situation "described" in the problem gives a nice math model.

Turning back to some of problems of math education today, we can recall one essential point. Math students today need to see numbers at work in the world they live in, if only to answer their persistent question: Why do I have to learn this? But it can be both difficult and time-consuming to find appropriate school-level problems that demonstrate how people actually use mathematical thinking in concrete settings. As we can see from the first example of Jinkoki book this problem has a natural solution in the approach used by math teachers in samurai classes.

In the Edo period people used colza oil for lighting their homes. The following problem is one typical example.

Exercise 2. *(Oil distribution problem) A colza - oil peddler is hawking oil. One evening on the way home, a customer asks him for 5 sho*[2] *of oil. But the oil peddler only has 10 sho of oil left in his big tub, and no way to measure out oil except two empty ladles that can hold 3 and 7 sho. How does the oil peddler sure out five colza for the customer?*

Since the solution is similar to the previous one we skip it and leave to the reader. However, possible modification that needs some creative argument to find the solution is the following one.

Exercise 3. *(Oil distribution problem modified) We have a full pot of oil. The pot has 10 sho capacity. How can we divide the oil into halves by using only the pot, a 3 sho cup and a 7 sho cup?*

This is one possible way to try to implement history of mathematics in math teaching and in the same way to have creative content of the work in the course.

If we recall Flash game associated with the Diophantine equation

$$Bx = A + Cy, \tag{1}$$

(see Figure 2 or go directly to the game) we can generalize the Oil distribution problem and try the following tasks:

- modify the problem so that the Flash game can be used as a solution

[1] A tan is a unit for measuring a bolt of cloth about 34 cm wide. One tan of such a cloth is about 10 m.
[2] 1 sho = 1.8 liter. Note: The Japanese sho differs from the Chinese sho.

- generalize the Oil distribution problem and find an algorithm for solution that can be solved independently; of the concrete data;
- modify the Flash software and adapt it to the previous task.

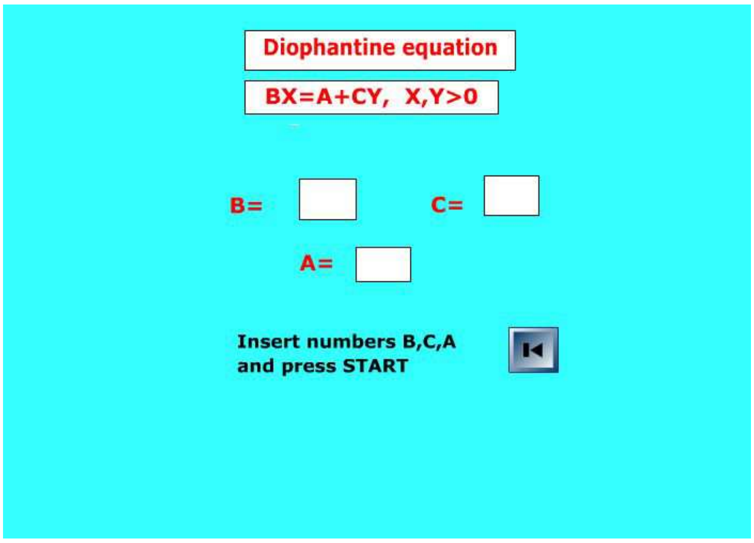

Figure 2: Game with Diophantine equation

We close this short walk in the History of Japanese Mathematics by proposing an example for further applications.

Exercise 4. *A rich man had an old and new wives who each has 15 children. Now, one of the children can get a right to receive their father's treasures by the game as follows. "All the children make a circle. Start counting them from a child and 10th child should drop off. Then, start from the next one, and continue it. The last one will get a right." Try to find strategy so that the first son of the old wife will win, if he has the right to show the starting point of the game.*

Exercise 5. *Some thieves stole some silk cloths and now they are trying to divide them. If each one has 8 cloths, the lack is 7. If each one has 7 cloths, the remainder is 8. How many are the thieves and cloths ?*

References

[1] Georgiev, V., Mushkarov, O., Ulovec, A., Dimitrova, N., Mogensen, A., Sendova, E. MEETING in Mathematics, Demetra Publishing House, Sofia, 2008

[2] A. Ulovec, J.Anderson, S. Čeretková, N. Dimitrova, V. Georgiev, O.Mushkarov E Sendova, *MATH to EARTH* 2010.

[3] Kajikawa, *....needs translation from Japanese .....*

[4] J. Šunderlk and E. Barcková, *Best spot - investigation with circles*, chapter in this book.

[5] M. Yoshida (revised and commented by S. Ohya) "Jinkoki", Iwanami Shoten, Japan, 1977.