Comenius Dynamat

Meeting Reykjavik 2013

Problem posing Lab Pisa(after the project of Marco Ferrigo)

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The team preparing the ebook is not homogeneous and it is composed by specialists in didactics of Mathematics as well as pure mathematicians without long experience in didactics. This particular point seems to become very useful since real interaction between different fields can give very positive impact on the whole work(but can provoke also negative results also!).

There were no particular restrictive schemes imposed at the beginning of the work.

1) The arguments in the didactic units had as an initial point models and events from the surrounding world

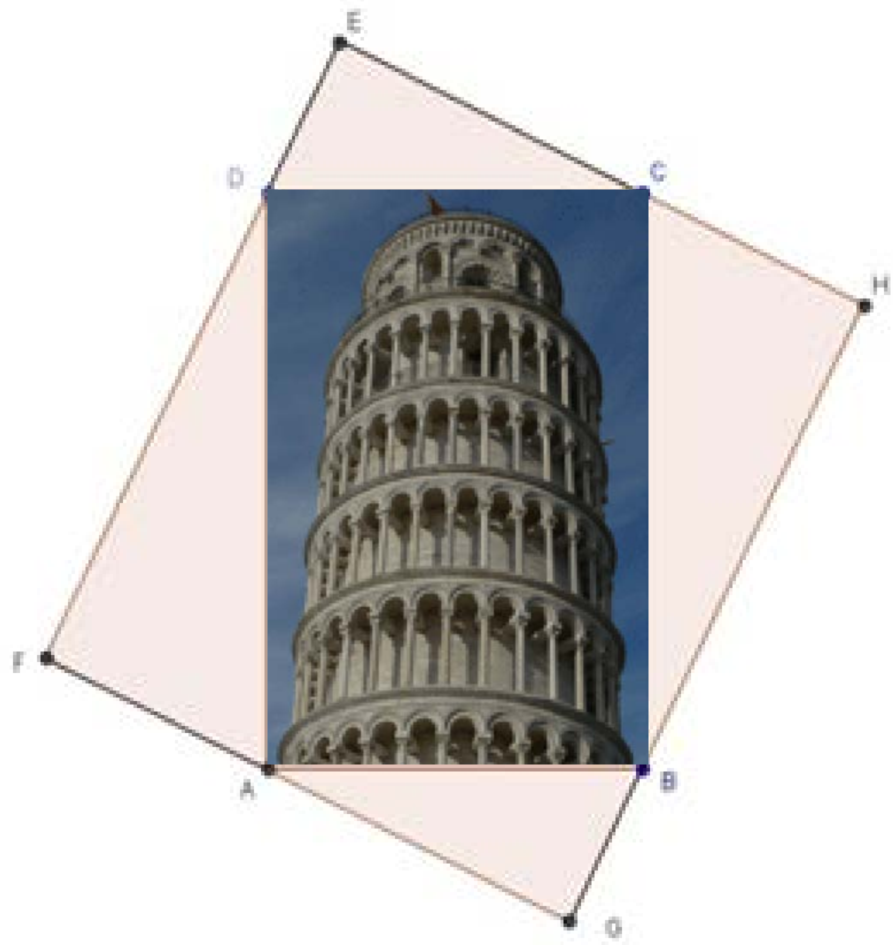
2)The arguments in the units have reasonable math contents supporting the preparation of future and in service teachers in Mathematics

3) We tried to unify all different arguments by using creative and attractive new ideas in posing, solving math problems and teaching math

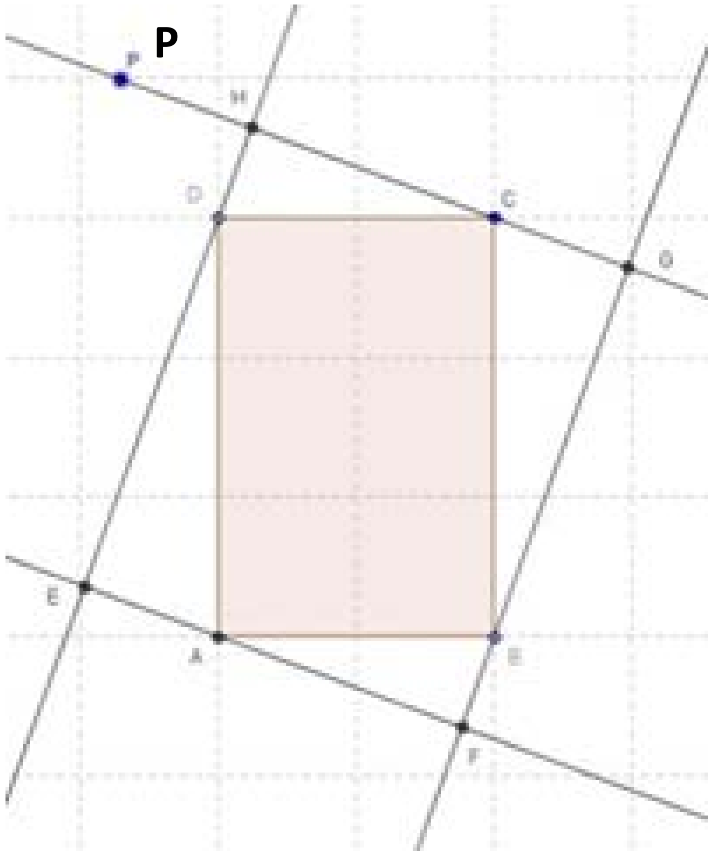
We will try to describe how easily a well-driven student can find by himself new problems and attempts to generalize or modify the problem given, having a concrete experience of investigation in mathematics.

DESCRIPTION OF THE PROBLEM

Given a rectangle ABCD, find, if it exists, a rectangle EFGH circumscribed to the previous one, such that the area of the EFGH is twice the area of ABCD.



A good way to attack the problem is by considering it in GeoGebra. Drawing perpendicular lines passing for each point A,B,C,D, we can represent the problem as explained in Fig. 3: the only free parameter is the position of point P:



Point P is the unknown one

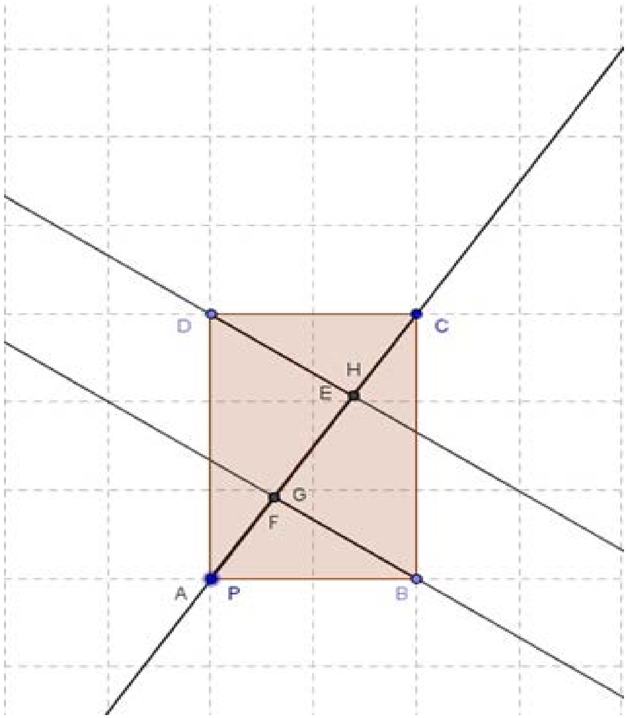
Through the exploration of the problem via GeoGebra, a student can find a good way to conjecture that a solution always exists, and an hypothesis of how to find it. The main point is to help the student to verify whether his/her intuition is correct or not, by formal demonstration. This way we have a possibility to emphasize the importance of both the intuitive and formal part of mathematics.

The perfect situation arises when some students have "wrong" intuitions (i.e. that are not correct, or that do not lead to the solution) and other students have "right" intuitions (i.e. that are correct and lead to the solution).

A good teacher should analyze deeply any suggest from his/her students, in order to help them comprehend why they are wrong, or to fully comprehend why they are right.

Anyway, we can present two possible ways to find a solution.

Solution by intuition: with the help of GeoGebra, one could decide to see what happens if the point P is put inside the rectangle ABCD: if P coincides with A, we find the inclination we want.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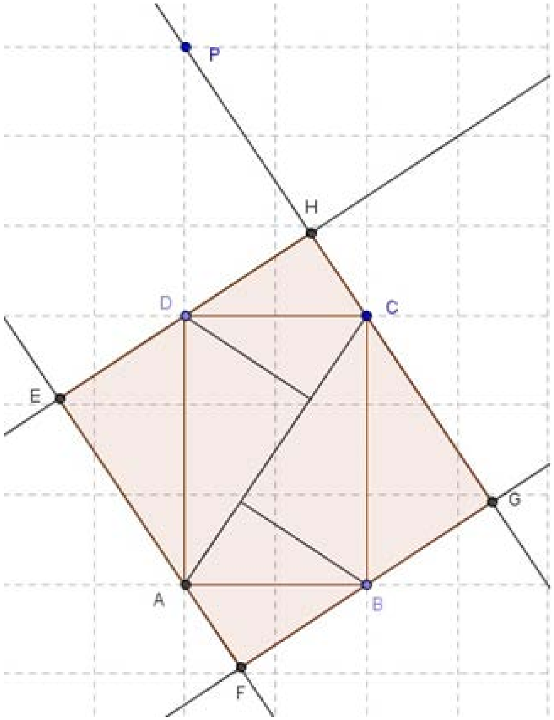
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Solution with help of other tools: using GeoGebra at its full potential, we can create a spreadsheet that calculates the area of our rectangles. Moving the point P, we find that at some points the area is very small (even 0), while it easily assumes values greater than double the area of ABCD. One can therefore guess where to put the point P to obtain its solution, and then try to prove it works.

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Some problems that may arise (or that should be proposed) during the investigation are here described:

What rectangle EFGH has a maximum area? Especially when using a spreadsheet, a student can guess on how to build it. There are many ways to see that maximum area is obtained when considering a square circumscribed: a nice proof starts from noticing that the vertices move along semicircles, and showing that the maximum area is obtained when the angles formed are of $45°$.

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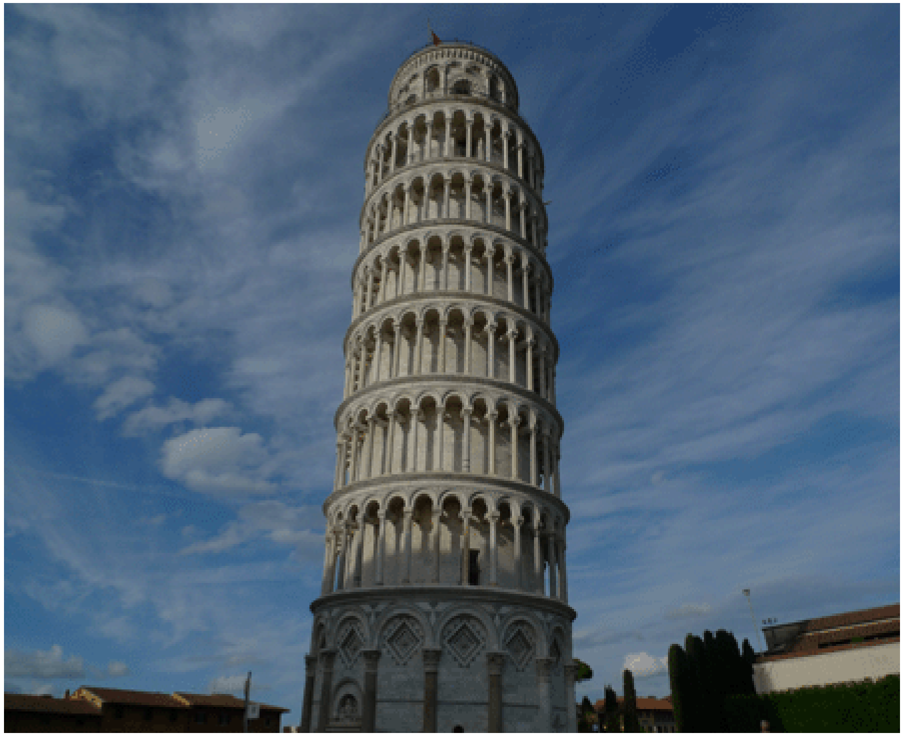

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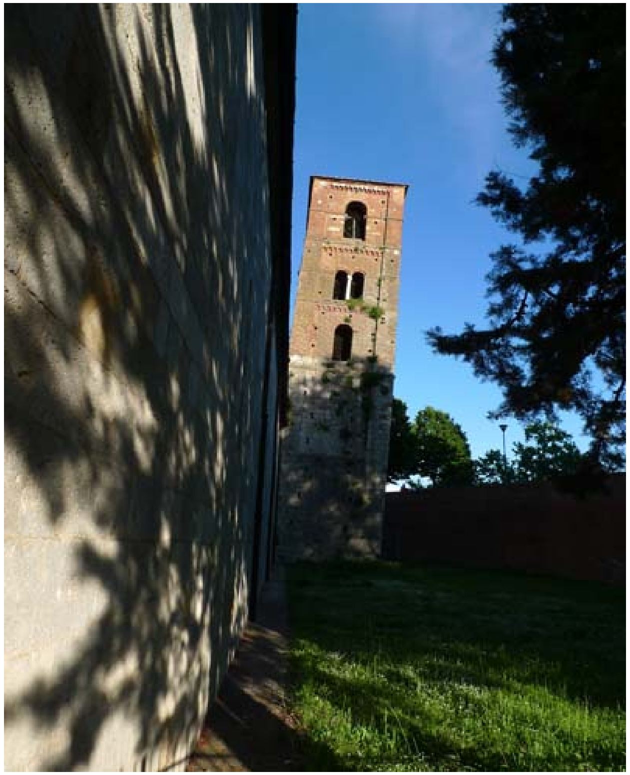

IT is really pendent