Dynamical simulation of stochastic phenomena using Excel

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Developing a sense of concepts from probability and statistics and the connection between the two can be a quite demanding struggle.

In this chapter I will demonstrate how to use spreadsheets to simulate situations which can add to experiences in the topics and thereby enhance understanding.

Flipping a coin

This may be one of the most common stochastic experiments in the world although people usually don't think of it as an experiment. Only mathematicians think of it in this way. Chances of getting heads or tails are to most people naturally put to fifty-fifty. But what does it mean to say that the probability of getting head is 50% or 0.5? Does it mean that you get a head and a tail whenever you toss a coin twice? Certainly not as you can immediately see if you try i out. Doing some statistics can clear things up a bit.

Tossing a coin repeatedly keeping track of the number of heads can be very informative and if you have never tried it you should offer the next half an hour to such an experiment.

Here I am going to use Excel to simulate the tossing.

In Excel there is a function named RANDBETWEEN which by use of random number generator can simulate the flipping of a coin and many other stochastic experiments.

Let's immediately see how it works and what it can do for us.

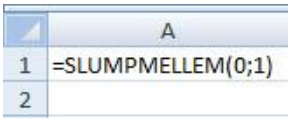

Fig. 1 Formula simulating the tossing of a coin. The Danish word SLUMPMELLEM corresponds to the English RANDBETWEEN. HEAD = 1. TAIL = 0.

If you write RANDBETWEEN(a;b) you get randomly chosen numbers from the set {a,a+1,.....,b-1,b}. Be aware that in different languages are used different delimiters – usually ";" or ","

If you fill in this function in a cell you will get randomly picked 0 or 1 each time the spreadsheet is calculated, e.g. when you press F9 function key.

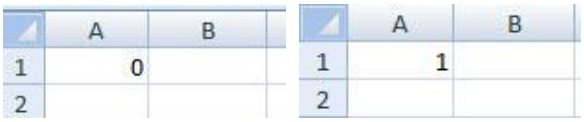

Fig. 2 Pressing F9 resultetd in these two outcomes

If you want to simulate 100 flips with a coin (or one flip with 100 coins) you can have it

	A	B	C	D	E	F	G	H	I	J
1	1	0	0	1	0	1	1	1	1	1
2	0	1	1	0	0	1	0	1	0	0
3	1	0	1	1	0	0	0	1	1	1
4	1	0	1	0	0	1	0	0	1	0
5	0	0	1	0	1	1	0	0	0	0
6	1	1	1	0	1	0	0	0	1	1
7	1	0	0	0	0	1	1	0	1	1
8	0	0	1	0	1	1	1	0	0	1
9	0	1	1	0	1	0	1	0	1	1
10	0	0	0	1	1	0	0	1	0	0

Fig. 3 100 flips with our 0/1 coin

You can do it by copying the RANDBETWEEN function to the relevant cells. Each copy functions independently of each other.

	A	B	C	D	E
1	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)
2	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)
3	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)
4	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)
5	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)
6	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)
7	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)
8	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)
9	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)
10	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)	=SLUMPMELLEM(0;1)
11					

Fig. 4 Independent copies of SLUMPMMELLEM = RANDBETWEEN

Next you need to count heads and tails. There is a function named COUNTIF for this purpose.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	1	0	0	0	1	0	0	0	1	0		No. of heads (1)	No. of tails (0)
2	0	0	0	1	0	0	0	0	0	1		45	55
3	0	0	1	1	1	0	0	0	1	1			
4	0	1	1	0	0	0	0	1	1	1			
5	0	1	0	0	0	1	1	0	1	0			
6	1	1	0	0	0	1	1	0	0	1			
7	1	0	1	1	1	0	1	0	1	0			
8	0	1	1	1	0	0	0	0	1	1			
9	0	1	0	1	1	0	0	1	1	0			
10	1	0	0	1	0	0	0	1	1	1			

Fig. 5 Heads and tails are counted by COUNTIF function and results are in cells L2 and M2

	L	M
	No. of heads (1)	No. of tails (0)
	=TÆL.HVIS($A$1:$J$10;1)	=TÆL.HVIS($A$1:$J$10;0)

Fig. 6 Formulas corresponding to Fig. 5.In Danish the function COUNTIF is named TÆL.HVIS

L	M
No. of heads (1)	No. of tails (0)
56	44

L	M
No. of heads (1)	No. of tails (0)
50	50

Fig. 7 Pressing F9 reflips the coins.

You can get an equal number of heads and tails but it's not what normally happens. Try i out and se how often i happens in e.g. 50 sets of 100 flips.

Perhaps you would like to do a long series of flips and keep track of the proportion of heads as the number grows. Something like shown in Fig. 8 may be interesting to examine.

	A	B	C	D
1	Flip no. n	Face up	Heads up to flip no. n	Fraction of heads up to flip no. n
2	1	1	1	1,000000
3	2	0	1	0,500000
4	3	1	2	0,666667
5	4	1	3	0,750000
6	5	1	4	0,800000
7	6	1	5	0,833333
8	7	1	6	0,857143
9	8	0	6	0,750000
10	9	0	6	0,666667
11	10	1	7	0,700000
12	11	1	8	0,727273
13	12	0	8	0.666667

Fig. 8 By copying downwards you can make long series of tossing a coin. For formulas se Fig. 9

	A	B	C	D
1	Flip no. n	Face up	Heads up to flip no. n	Fraction of heads up to flip no. n
2	1	=SLUMPMELLEM(0;1)	=TÆL.HVIS($B$2:B2;1)	=C2/A2
3	=A2+1	=SLUMPMELLEM(0;1)	=TÆL.HVIS($B$2:B3;1)	=C3/A3
4	=A3+1	=SLUMPMELLEM(0;1)	=TÆL.HVIS($B$2:B4;1)	=C4/A4
5	=A4+1	=SLUMPMELLEM(0;1)	=TÆL.HVIS($B$2:B5;1)	=C5/A5
6	=A5+1	=SLUMPMELLEM(0;1)	=TÆL.HVIS($B$2:B6;1)	=C6/A6
7	=A6+1	=SLUMPMELLEM(0;1)	=TÆL.HVIS($B$2:B7;1)	=C7/A7
8	=A7+1	=SLUMPMELLEM(0;1)	=TÆL.HVIS($B$2:B8;1)	=C8/A8
9	=A8+1	=SLUMPMELLEM(0;1)	=TÆL.HVIS($B$2:B9;1)	=C9/A9
10	=A9+1	=SLUMPMELLEM(0;1)	=TÆL.HVIS($B$2:B10;1)	=C10/A10
11	=A10+1	=SLUMPMELLEM(0;1)	=TÆL.HVIS($B$2:B11;1)	=C11/A11
12	=A11+1	=SLUMPMELLEM(0;1)	=TÆL.HVIS($B$2:B12;1)	=C12/A12
13	=A12+1	=SLUMPMELLEM(0;1)	=TÆL.HVIS($B$2:B13;1)	=C13/A13

Fig. 9 Formulas corresponding to Fig. 8

A useful graphical representation is a xy dot-plot shoving fraction of heads as a function of n.

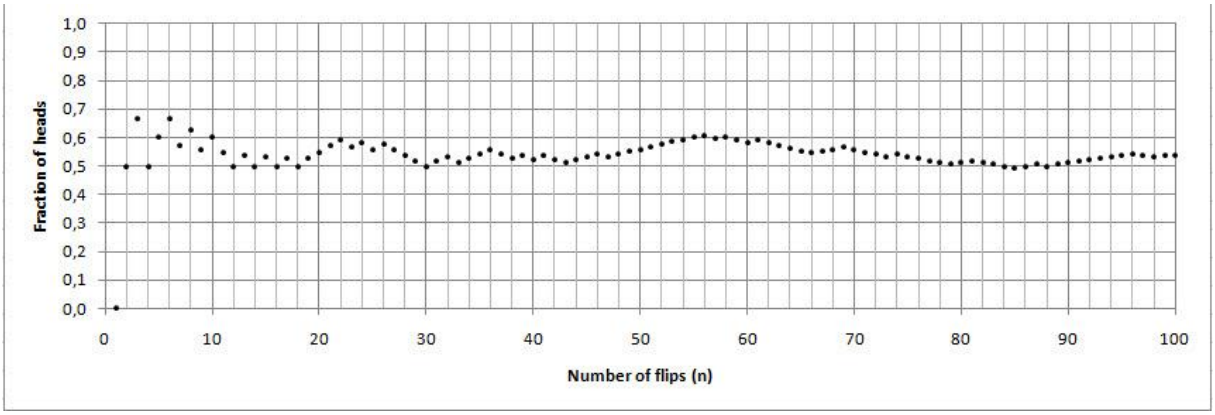

Fig. 10 Fraction of heads up to n tosses as a function of n

Although the graph only has points at discrete values of the variable n it can help the eyes to draw lines between the points

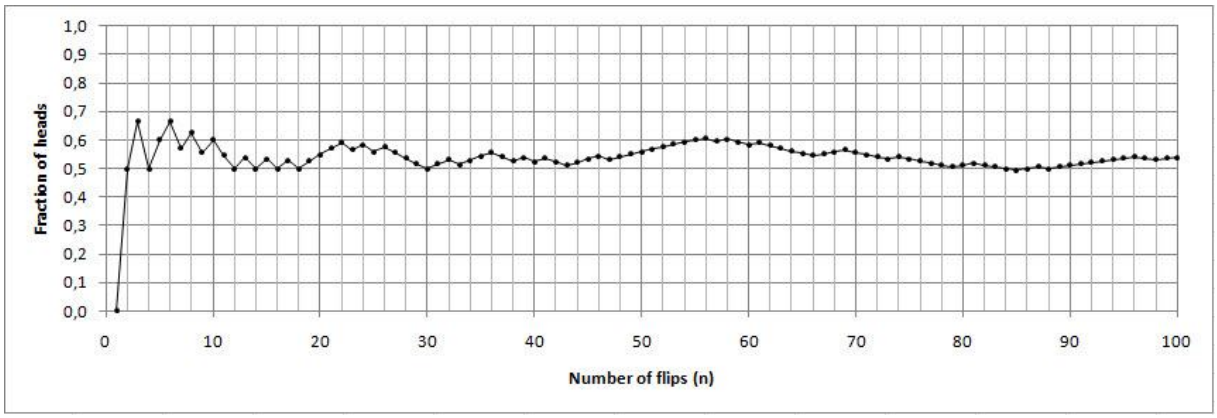

Fig. 11 Piecevise graphics to improve visualisation

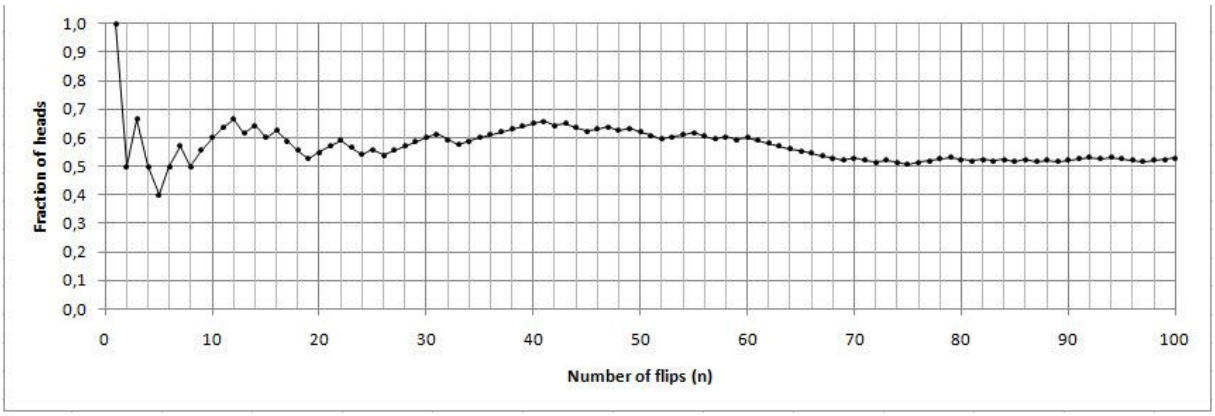

Fig. 12 Pressing F9 gives another sequence of 100 flips.

It's possible to make a much longer sequence of flips. You copy downwards in fig. 8 & 9- In my version of Excel there are 2^{20} rows to fill up so perhaps you run out of memory before you run out of rows. Below you see to sequences with 1000 flips

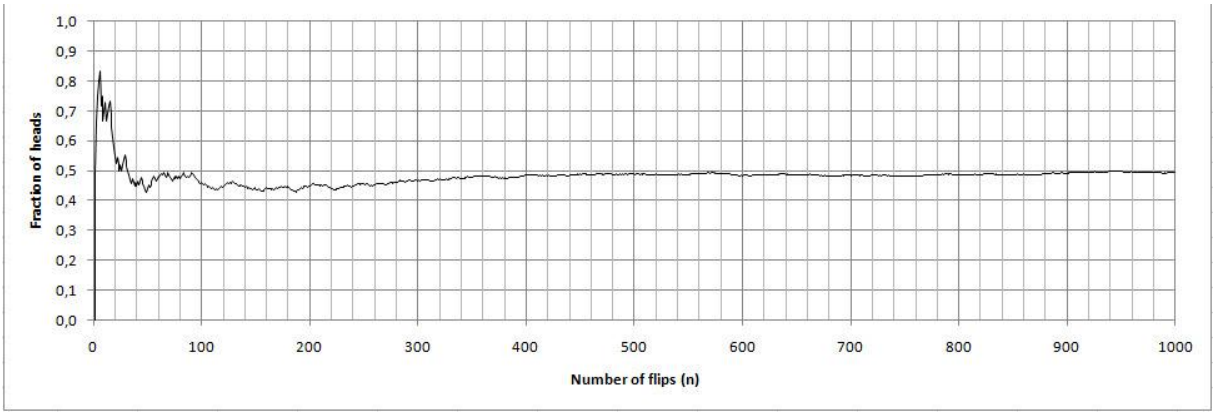

Fig. 13 Relative frequency as a function of n for 1000 flips of a simulated coin

If you want 10000 flips it is no problem. Copy downwards and you get it.

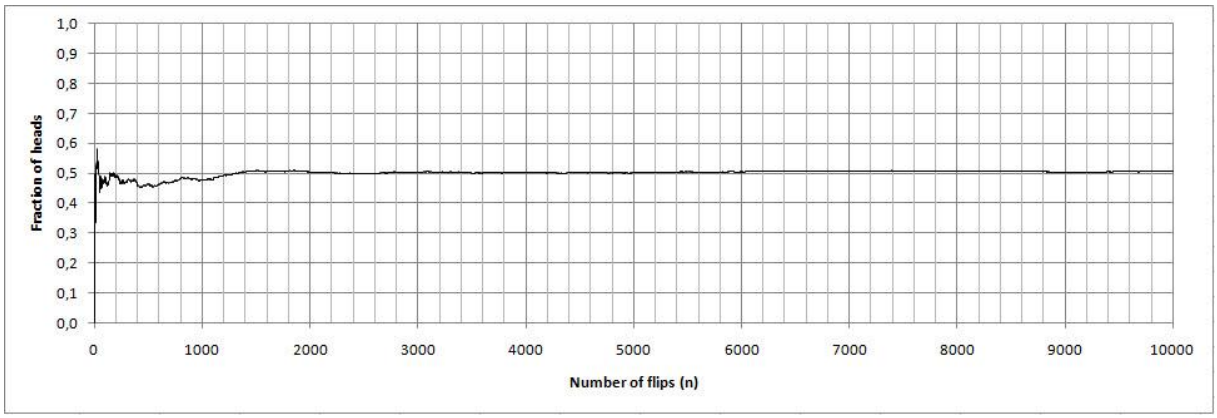

Fig. 14 Relative frequency as a function of n for 10000 flips of a simulated coin

Conclusions from the experiments above suggest that the limiting value of the relative frequencies equals the probability. This assertion hints at the so called law of large numbers. To explore this further in the next section the tossing of five dices are examined.

Tossing five dices

Fig. 15 Five dices

You toss five dices and count the number showing six eyes. It can be any integer between 0 and 5.

Tossing again and again keeping track of the number of 6-faces after e.g. 50 rolls you will have data to fill in a table as shown in Fig. 16.

k	No. of rolls with k dices showing 6	Frequen-ces of rolls with k dices showing 6
0	21	0,4200
1	19	0,3800
2	9	0,1800
3	1	0,0200
4	0	0,0000
5	0	0,0000

Fig. 16 Statistics from 50 rolls of five dices

The table in Fig 16 shows that out of the 50 rolls 21 resulted in none of the five dices turning face 6 upwards. Dividing 21 by 50 you calculate the proportion or relative frequency as it is called.

The table is part of a spreadsheet designed to dynamically follow the process as you advance the number of rolls by one.

	A	B	C	D	E
1	**Reset (0/1)**				
2	1				
3	**No. of rolls**				
4	50				
5	**D1**	**D2**	**D3**	**D4**	**D5**
6	3	4	6	3	1
7	**No. of dices showing 6**				
8	1				
9	**k**	**No. of rolls with k dices showing 6**	**Frequen-ces of rolls with k dices showing 6**	**Binomial distri-bution**	
10	0	21	0,4200	0,4019	
11	1	19	0,3800	0,4019	
12	2	9	0,1800	0,1608	
13	3	1	0,0200	0,0322	
14	4	0	0,0000	0,0032	
15	5	0	0,0000	0,0001	

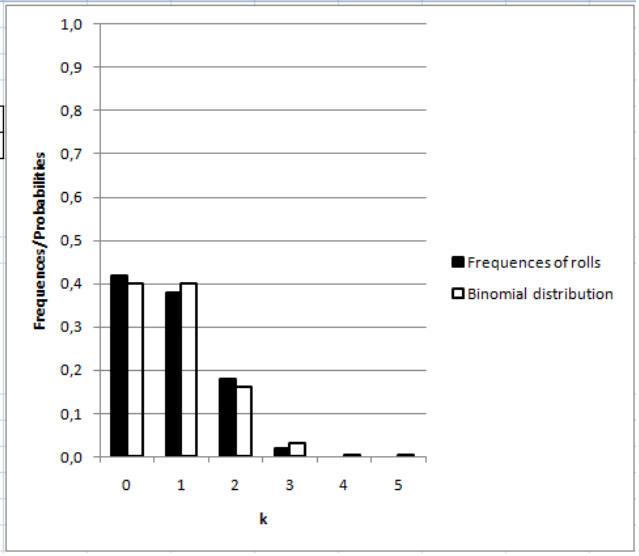

Fig. 17 Dynamical simulation of tossing 5 dices. Pressing F9 generates a new roll and data are acumulated as one proceeds.

	A	B	C	D	E
1	**Reset (0/1)**				
2	1				
3	**No. of rolls**				
4	=HVIS(A2=1;A4+1;0)				
5	**D1**	**D2**	**D3**	**D4**	**D5**
6	=SLUMPMELLEM(1;6)	=SLUMPMELLEM(1;6)	=SLUMPMELLEM(1;6)	=SLUMPMELLEM(1;6)	=SLUMPMELLEM(1;6)
7	**No. of dices showing 6**				
8	=HVIS(A2=1;TÆL.HVIS(A6:E6;6);"")				
9	**k**	**No. of rolls with k dices showing 6**	**Frequen-ces of rolls with k dices showing 6**	**Binomial distri-bution**	
10	0	=HVIS($A$2=1;HVIS($A$8=A10;B10+1;B10);0)	=HVIS($A$4>0;B10/$A$4;0)	=BINOMIALFORDELING(A10;5;1/6;0)	
11	1	=HVIS($A$2=1;HVIS($A$8=A11;B11+1;B11);0)	=HVIS($A$4>0;B11/$A$4;0)	=BINOMIALFORDELING(A11;5;1/6;0)	
12	2	=HVIS($A$2=1;HVIS($A$8=A12;B12+1;B12);0)	=HVIS($A$4>0;B12/$A$4;0)	=BINOMIALFORDELING(A12;5;1/6;0)	
13	3	=HVIS($A$2=1;HVIS($A$8=A13;B13+1;B13);0)	=HVIS($A$4>0;B13/$A$4;0)	=BINOMIALFORDELING(A13;5;1/6;0)	
14	4	=HVIS($A$2=1;HVIS($A$8=A14;B14+1;B14);0)	=HVIS($A$4>0;B14/$A$4;0)	=BINOMIALFORDELING(A14;5;1/6;0)	
15	5	=HVIS($A$2=1;HVIS($A$8=A15;B15+1;B15);0)	=HVIS($A$4>0;B15/$A$4;0)	=BINOMIALFORDELING(A15;5;1/6;0)	

Fig. 18 Formulas corresponding to the spreadsheet in Fig. 17. Please zoom in to see the details.

HVIS is Danish for the function IF.

The spreadsheet makes use of circular references which you have to set the calculation mode to Manual as shown in Fig. 18

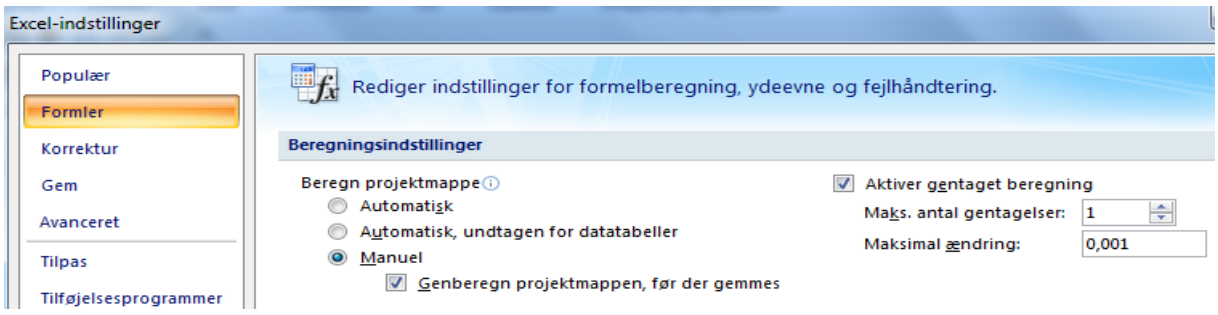

Fig. 19 Setting the properties for Formulas/Calculations to manual whith 1 repetition.

Guide to the spreadsheet in Fig. 17 and 18

You reset the spreadsheet by entering 0 in cell A2 and press F9. For more details regarding these spread sheet detail see section 4.3.2 in [1].

	A	B	C	D	E
1	Reset (0/1)				
2	0				
3	No. of rolls				
4	0				
5	D1	D2	D3	D4	D5
6	3	6	3	5	5
7	No. of dices showing 6				
8					
9	k	No. of rolls with k dices showing 6	Frequen-ces of rolls with k dices showing 6	Binomial distri-bution	
10	0	0	0,0000	0,4019	
11	1	0	0,0000	0,4019	
12	2	0	0,0000	0,1608	
13	3	0	0,0000	0,0322	
14	4	0	0,0000	0,0032	
15	5	0	0,0000	0,0001	

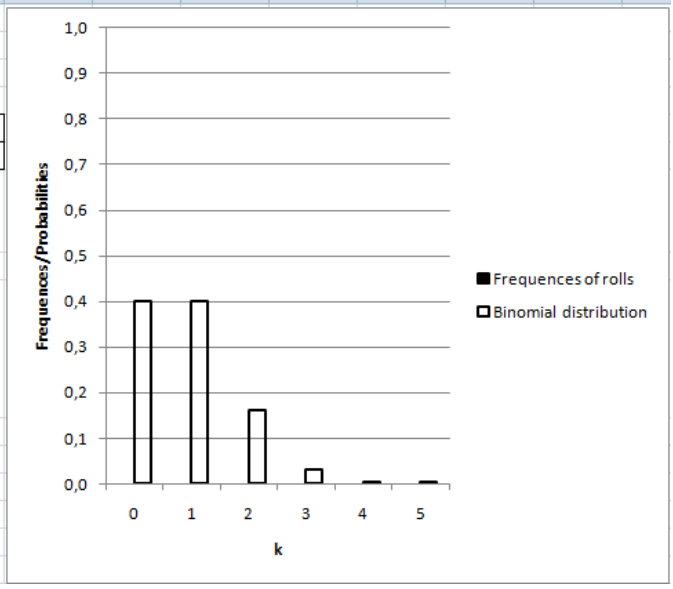

Fig. 20 Result of resetting the spreadsheet

Then you enter 1 in cell A2 and press F9 and you have the first roll.

	A	B	C	D	E
1	Reset (0/1)				
2	1				
3	No. of rolls				
4	1				
5	D1	D2	D3	D4	D5
6	5	3	4	2	1
7	No. of dices showing 6				
8	0				
9	k	No. of rolls with k dices showing 6	Frequen-ces of rolls with k dices showing 6	Binomial distri-bution	
10	0	1	1,0000	0,4019	
11	1	0	0,0000	0,4019	
12	2	0	0,0000	0,1608	
13	3	0	0,0000	0,0322	
14	4	0	0,0000	0,0032	
15	5	0	0,0000	0,0001	

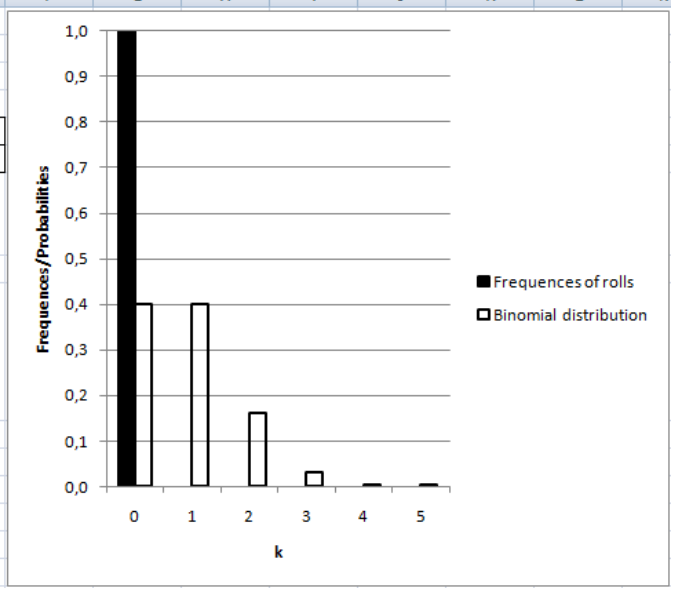

Fig. 21 Result after first roll.

Pressing F9 again and again you will have more and more rolls. The statistics and the graphic are dynamically updated after each roll. After e.g. 25 rolls you may end up with something as in Fig. 22.

	A	B	C	D	E
1	Reset (0/1)				
2	1				
3	No. of rolls				
4	25				
5	D1	D2	D3	D4	D5
6	5	5	4	3	3
7	No. of dices showing 6				
8	0				
9	k	No. of rolls with k dices showing 6	Frequen-ces of rolls with k dices showing 6	Binomial distri-bution	
10	0	12	0,4800	0,4019	
11	1	7	0,2800	0,4019	
12	2	6	0,2400	0,1608	
13	3	0	0,0000	0,0322	
14	4	0	0,0000	0,0032	
15	5	0	0,0000	0,0001	

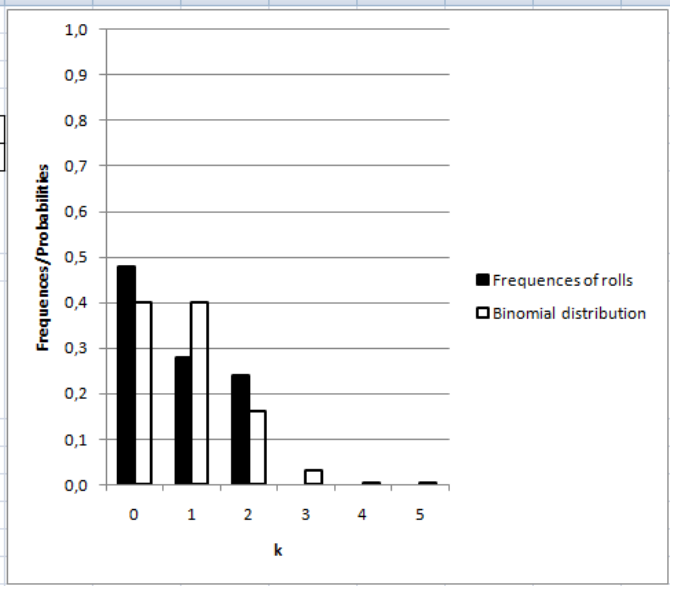

Fig. 22 Result after 25 rolls

The main point is that the relative frequencies steadies as you do more and more tosses. After 1125 rolls the graphics looked like Fig. 23.

	A	B	C	D	E
1	Reset (0/1)				
2	1				
3	No. of rolls				
4	1125				
5	D1	D2	D3	D4	D5
6	6	1	3	2	6
7	No. of dices showing 6				
8	2				
9	k	No. of rolls with k dices showing 6	Frequen-ces of rolls with k dices showing 6	Binomial distri-bution	
10	0	448	0,3982	0,4019	
11	1	460	0,4089	0,4019	
12	2	186	0,1653	0,1608	
13	3	29	0,0258	0,0322	
14	4	2	0,0018	0,0032	
15	5	0	0,0000	0,0001	

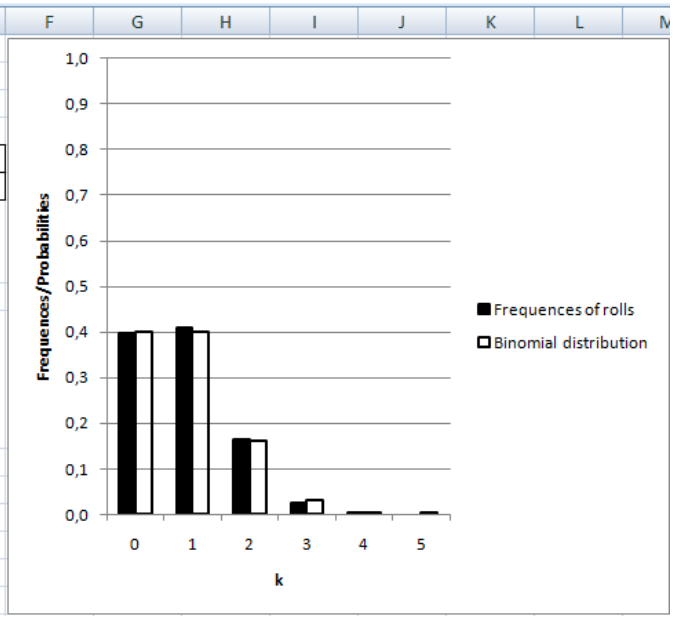

Fig. 23 Result after 1125 rolls

The black bars show the relative frequencies while the white bars show probabilities calculated by means of the so called binomial distribution which is the theoretical model for the situation we are examining.

As in the tossing of a coin section you experience the probabilities as limiting values of relative frequencies as the number of rolls tends to infinity.

Static graphical representation of a long sequence of rolls

In this last section the rolling of the dices are treated in a way similar to what was done to the tossing of the coin. The price paid for this is that many more cells have to be filled since we need to keep al rolls in the spreadsheet. Each roll has to have its own row.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Roll no.	Dices					Number of 6	(Number of X = k up to n)/n, k = 0, 1, 2, 3, 4, 5					
2	n	D1	D2	D3	D4	D5	X	0,0000	1,0000	2,0000	3,0000	4,0000	5,0000
3	1	4	2	3	1	4	0	1,0000	0,0000	0,0000	0,0000	0,0000	0,0000
4	2	4	1	1	5	1	0	1,0000	0,0000	0,0000	0,0000	0,0000	0,0000
5	3	6	6	3	6	2	3	0,6667	0,0000	0,0000	0,3333	0,0000	0,0000
6	4	6	2	2	5	2	1	0,5000	0,2500	0,0000	0,2500	0,0000	0,0000
7	5	5	6	1	4	6	2	0,4000	0,2000	0,2000	0,2000	0,0000	0,0000
8	6	5	2	6	6	6	3	0,3333	0,1667	0,1667	0,3333	0,0000	0,0000
9	7	5	5	6	2	5	1	0,2857	0,2857	0,1429	0,2857	0,0000	0,0000
10	8	4	3	5	2	6	1	0,2500	0,3750	0,1250	0,2500	0,0000	0,0000
11	9	6	6	1	6	4	3	0,2222	0,3333	0,1111	0,3333	0,0000	0,0000
12	10	5	5	3	5	6	1	0,2000	0,4000	0,1000	0,3000	0,0000	0,0000
13	11	1	3	3	2	5	0	0,2727	0,3636	0,0909	0,2727	0,0000	0,0000
14	12	1	3	1	1	3	0	0,3333	0,3333	0,0833	0,2500	0,0000	0,0000
15	13	5	2	4	4	6	1	0,3077	0,3846	0,0769	0,2308	0,0000	0,0000
16	14	2	6	3	1	2	1	0,2857	0,4286	0,0714	0,2143	0,0000	0,0000

Fig. 24 The first 16 rows of the spreadsheet for graphing rolls of 5 dices

	A	B	C	D	E	F	G	H	I	J	K
1	Roll no.		Dices				Number of 6		(Number of X = k up to n)/n, k	= 0, 1, 2, 3, 4, 5	
2	n	D1	D2	D3	D4	D5	X	0	1	2	3
3	1	=SLUMPMELLEM(1;6)	=SLUM	=SLUM	=SLUM	=SLUM	=TÆL.HVIS(B3:F3;6)	=TÆL.HVIS($G$3:$G3;H$2)/$A3	=TÆL.HVIS($G$3:$G3;I$2)/$A3	=TÆL.HVIS($G$3:	=TÆL.HVIS($G$3:
4	=A3+1	=SLUMPMELLEM(1;6)	=SLUM	=SLUM	=SLUM	=SLUM	=TÆL.HVIS(B4:F4;6)	=TÆL.HVIS($G$3:$G4;H$2)/$A4	=TÆL.HVIS($G$3:$G4;I$2)/$A4	=TÆL.HVIS($G$3:	=TÆL.HVIS($G$3:
5	=A4+1	=SLUMPMELLEM(1;6)	=SLUM	=SLUM	=SLUM	=SLUM	=TÆL.HVIS(B5:F5;6)	=TÆL.HVIS($G$3:$G5;H$2)/$A5	=TÆL.HVIS($G$3:$G5;I$2)/$A5	=TÆL.HVIS($G$3:	=TÆL.HVIS($G$3:
6	=A5+1	=SLUMPMELLEM(1;6)	=SLUM	=SLUM	=SLUM	=SLUM	=TÆL.HVIS(B6:F6;6)	=TÆL.HVIS($G$3:$G6;H$2)/$A6	=TÆL.HVIS($G$3:$G6;I$2)/$A6	=TÆL.HVIS($G$3:	=TÆL.HVIS($G$3:
7	=A6+1	=SLUMPMELLEM(1;6)	=SLUM	=SLUM	=SLUM	=SLUM	=TÆL.HVIS(B7:F7;6)	=TÆL.HVIS($G$3:$G7;H$2)/$A7	=TÆL.HVIS($G$3:$G7;I$2)/$A7	=TÆL.HVIS($G$3:	=TÆL.HVIS($G$3:
8	=A7+1	=SLUMPMELLEM(1;6)	=SLUM	=SLUM	=SLUM	=SLUM	=TÆL.HVIS(B8:F8;6)	=TÆL.HVIS($G$3:$G8;H$2)/$A8	=TÆL.HVIS($G$3:$G8;I$2)/$A8	=TÆL.HVIS($G$3:	=TÆL.HVIS($G$3:
9	=A8+1	=SLUMPMELLEM(1;6)	=SLUM	=SLUM	=SLUM	=SLUM	=TÆL.HVIS(B9:F9;6)	=TÆL.HVIS($G$3:$G9;H$2)/$A9	=TÆL.HVIS($G$3:$G9;I$2)/$A9	=TÆL.HVIS($G$3:	=TÆL.HVIS($G$3:
10	=A9+1	=SLUMPMELLEM(1;6)	=SLUM	=SLUM	=SLUM	=SLUM	=TÆL.HVIS(B10:F10;6)	=TÆL.HVIS($G$3:$G10;H$2)/$A10	=TÆL.HVIS($G$3:$G10;I$2)/$A10	=TÆL.HVIS($G$3:	=TÆL.HVIS($G$3:
11	=A10+1	=SLUMPMELLEM(1;6)	=SLUM	=SLUM	=SLUM	=SLUM	=TÆL.HVIS(B11:F11;6)	=TÆL.HVIS($G$3:$G11;H$2)/$A11	=TÆL.HVIS($G$3:$G11;I$2)/$A11	=TÆL.HVIS($G$3:	=TÆL.HVIS($G$3:
12	=A11+1	=SLUMPMELLEM(1;6)	=SLUM	=SLUM	=SLUM	=SLUM	=TÆL.HVIS(B12:F12;6)	=TÆL.HVIS($G$3:$G12;H$2)/$A12	=TÆL.HVIS($G$3:$G12;I$2)/$A12	=TÆL.HVIS($G$3:	=TÆL.HVIS($G$3:
13	=A12+1	=SLUMPMELLEM(1;6)	=SLUM	=SLUM	=SLUM	=SLUM	=TÆL.HVIS(B13:F13;6)	=TÆL.HVIS($G$3:$G13;H$2)/$A13	=TÆL.HVIS($G$3:$G13;I$2)/$A13	=TÆL.HVIS($G$3:	=TÆL.HVIS($G$3:
14	=A13+1	=SLUMPMELLEM(1;6)	=SLUM	=SLUM	=SLUM	=SLUM	=TÆL.HVIS(B14:F14;6)	=TÆL.HVIS($G$3:$G14;H$2)/$A14	=TÆL.HVIS($G$3:$G14;I$2)/$A14	=TÆL.HVIS($G$3:	=TÆL.HVIS($G$3:

Fig. 25 Formulas corresponding to Fig 24. Zoom in for details.

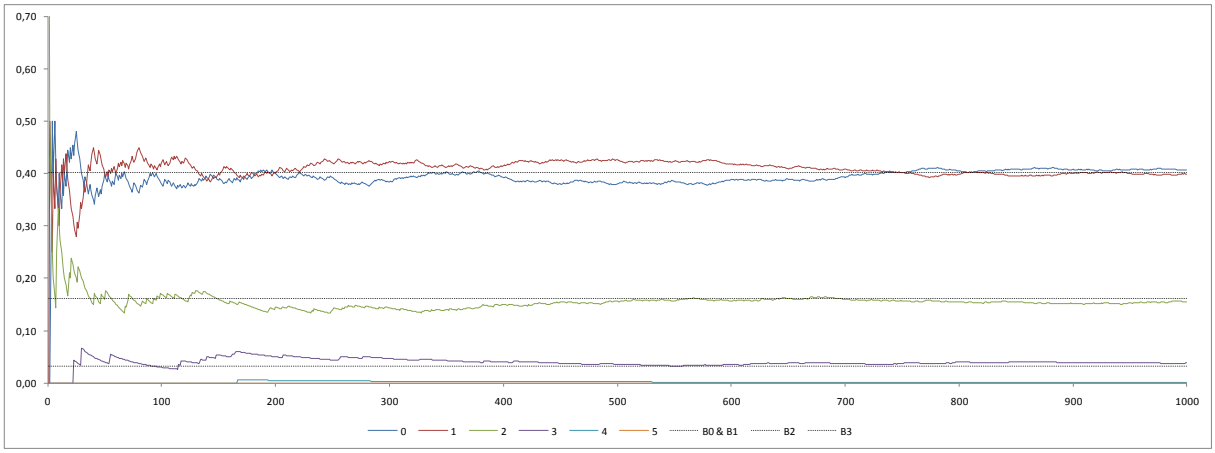

Fig. 26 xy-plot of relative frequencies of number of 6-faces in rolls with 5 dices as a funcion of the number of rolls.

In Fig. 26 we see another instance of giving evidence to the law of large numbers.

References

[1] http://www.math2earth.oriw.eu/publications/13_Animation.pdf (November, 2011)