GPS-geometry in the landscape

John Andersen

As described in [9] another chapter of this book GPS-technology makes it possible to track trips in the landscape. In this chapter I will try to turn it 180° around and determine which shapes I want as a route for my walk. The idea had been lurking in the background for a while, but during a jogging trip in Copenhagen I decided to go for it.

In Copenhagen you find the fortress Kastellet

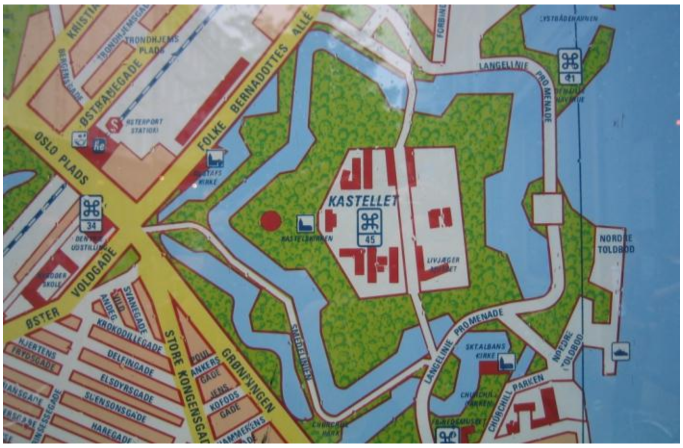


Fig. 1 Part of information map in the streets of Copenhagen

It is very renowned (at least in Denmark) and you see a lot of references to it. For instance you find it in the logo of The Association for the Beuatification of the Capital

Foreningen til Hovedstadens
FORSKØNNELSE

Fig. 2 Logo of The Associaton for the Beuatification of the Capital [3]

Fig. 3 Kastellet seen from above [4]

There is public acces to the fortress and the pathways around and it is a popular place for jogging. On my trips I tracked the route with my Garmin etrex VISTA HCx GPS and after uploading to Google Earth this star shaped geometrical object appeared

Fig. 4 Track from the author's jogging trip i Copenhagen on March 19th 2011

You can follow already made pathways in the landscape and track them with GPS and study the shapes. But what about the other way round: You start with som geometrical shape and then you plan a trip that will produce this shape on your GPS-device and on Google Earth.

1 Walking and GPS tracking geometrical shapes

At this place I could go through a lot of theory and produce a detailed manual for students to follow. I am not going to do this. As a teacher my job is to bring students challenges they have never met before and then learn from this. I have to expose myself to the unceartainty that comes with tasks I have no fixed idea where will lead me.

So lets leap right out in the middle of it to get som experiences from which to learn. [5]

One of the simplest geometrical shapes that comes to my mind is the equilateral triangle. So why not begin with this?

In a traditionel classroom you would take a sheet of paper to draw on. In this set up you need a fairly regular peace of land

Fig. 5 Industrial area still undeveloped serve as "schetchpad"

Then you must have an idea about how to produce a triangle in the landscape. Navigating do often include walking a ceartain distance in a certain direction. Then turning some specified angle and moving on in this new direction. Before the days of GPS a compass was used to manage the turning angles. GPS-devises also have built in compasses, so lets try it out.

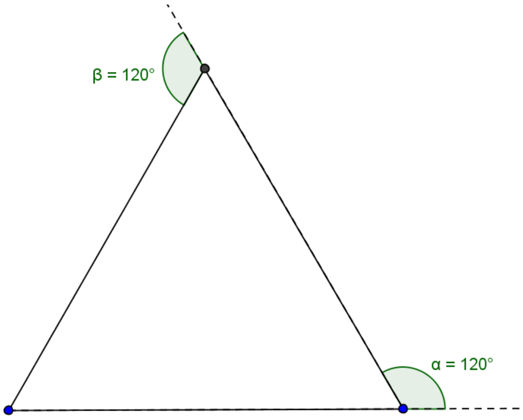


Fig. 6 Considerations on how to walk an equilateral triangle

I decide to walk 100 m in a fixed direction. Then turn 120° counterclockwise and move on for another 100 m. And then back to my starting point which i have stored in the GPS-devise as a so called "waypoint".

Fig. 7 Resulting track in the landscape shown by Google Earth

The result looks like a triangle but certainly not an equilateral one. Zooming do not make you happier

Fig. 8 Zooming in on "equilateral" triangle

This calls for more experinces so I decide to walk a regular pentagram recalling this figure

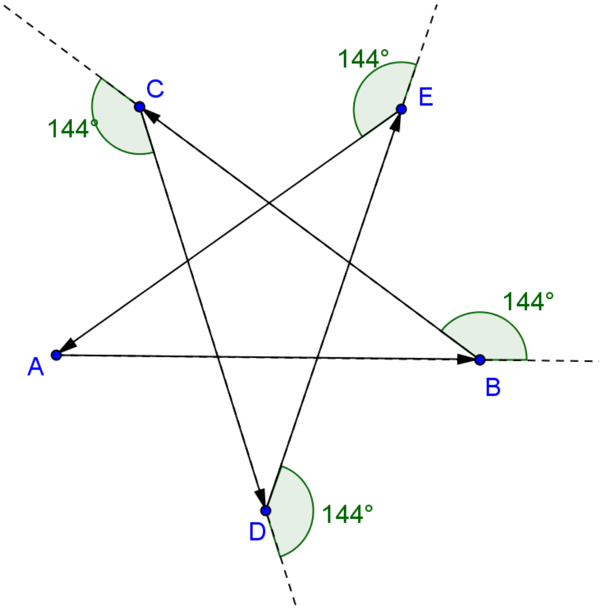


Fig. 9 Considerations on how to walk a regular pentagramme

Fig. 10 Pentagramme track uploaded to Google Earth

Since I had stored my starting position int the GPS I manage to find my way back to the starting point (the leftmost corner). And it is a pentagram but not regular. Looking carfully on Fig. 10 you see that I had to cross a road. This kind of geometry could be dangerous but luckely enough the traffic density was low at the time for performing the task.

I have already experienced that it is not at all easy to navigate precicely this way. I decide to do one more. Well, what about just a regular pentagon?

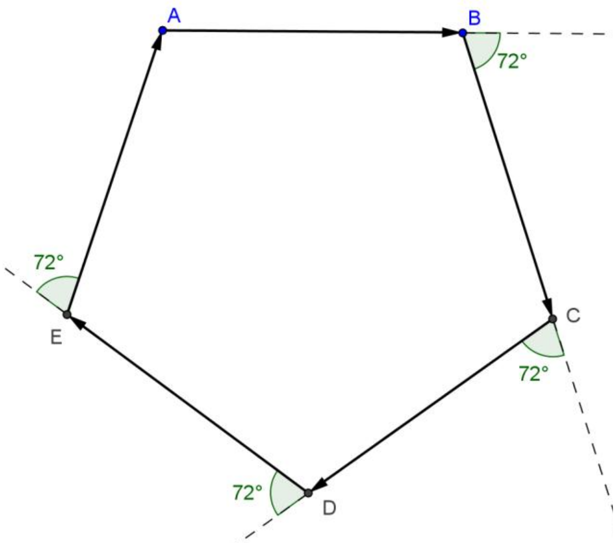


Fig. 11 Considerations on how to walk a regular pentagon

It turns out to be a regular disaster

Fig. 12 Irregular pentagon track in the fields

It must be the right time to return to study room and reflect on experiences i have had so far. Just in the spirit indicated by Kolb's Learning Cycle

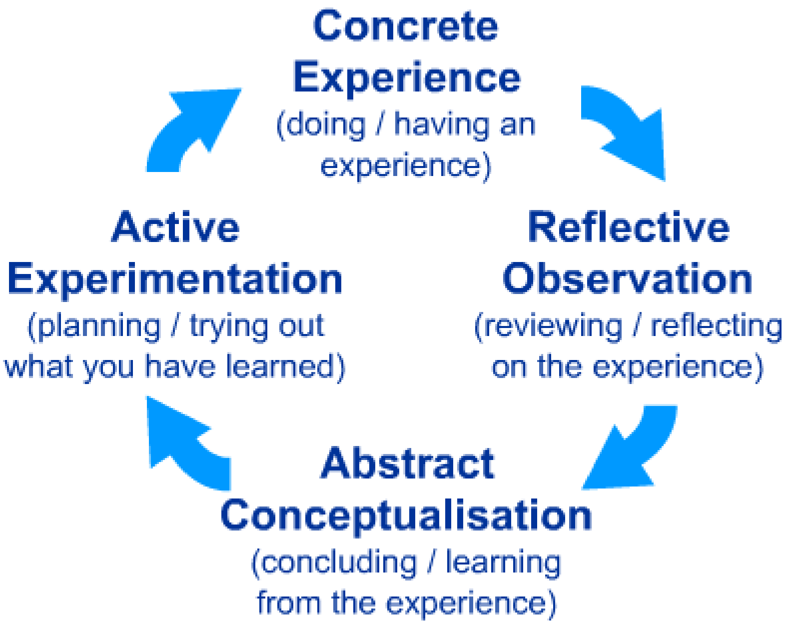


Fig. 13 Kolb's Learning Cykle [5]

It says in very short formulation: You have some concrete experience. In this case the shapes above. I have already done some reflective observation since I evaluated the figures to be not what was intended. Next part is to analyse what explanations can be given to the observations.

Importing the picture to GeoGebra measuring the sides and angles gives

Fig. 14 Track from Fig. 13 inserted in GeoGebra to be examined

One notices that the first four sides are of almost the same length. In the real world they were thought to be 100 meter. The reason why the last side is much to short is the fact that I stood much to near the starting point when I had finished the fourth side. In theory the interior angles all should measure 108°. The first angle at 102° is not good, but the real sin was committed at the 89° angle. This might be due to a mental miscalculation or a very unsteady handheld compass.

Now what can be done to improve performing?

It seems that trying to aim at a point using the GPS compass is much too inaccurate. But there is a function in my GPS called Project Waypoint. Here you can key in a fixed compass direction and a fixed distance and thereby create a waypoint at the desired position. Now you navigate to this waypoint and key in the next waypoint changing the direction for instance in accordance with Fig 11 if you want to track down a regular pentagon. The result is shown in Fig. 14a. On the right part of the picture the pentagon is compared to a yellow regular pentagon created with GeoGebra. Except for a minor deviation between points A and B it seems all right. In fact the walk from A to B taught me that aiming is not enough to ensure good results. You also have to monitor the tracking on the graphics interface of the GPS so that you can set in corrections as soon as possible.

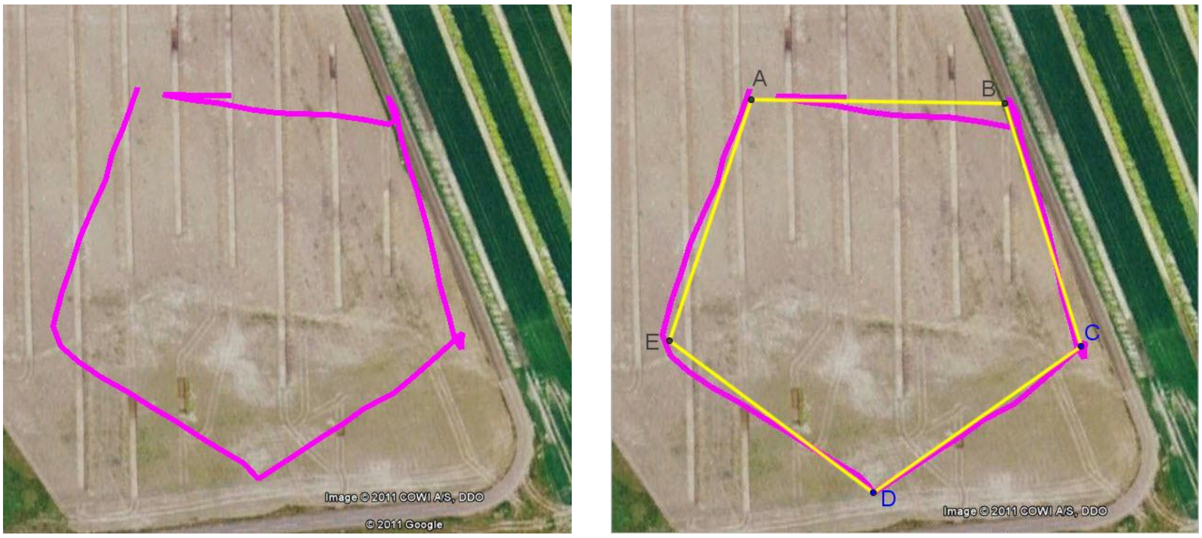


Fig. 15 Last regular pentagon tracking made using "Project Waypoint".

2 Using coordinates to mark waypoints and create routes.

Describing points on the surface of the Earth by coordinates is a rather complicated matter if you want to master all details. Most people have heard of lattitude and longitude. Today the growing interest in using GPS for navigating in e.g. traffic, sports and photography may gradually enlarge peoples knowledge of this subject matter. Perhaps it is also a relevant subject for teaching in mathematics lessons.

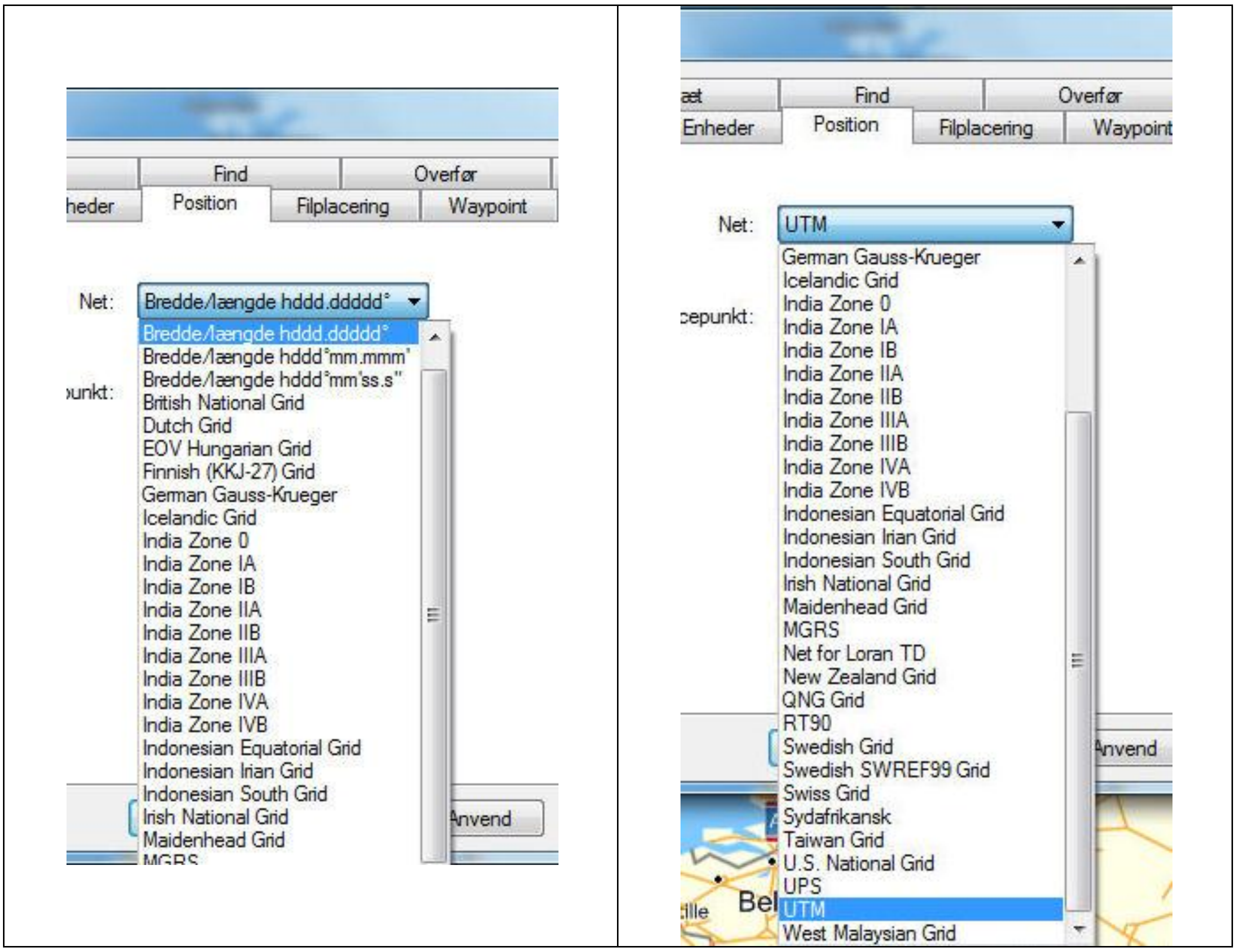


Fig. 16 In MapSource you can choose between lots of different coordinates for positions. Relevant for use in the actual case is lattitude – longitude (Bredde/længde in the menu above) and UTM.

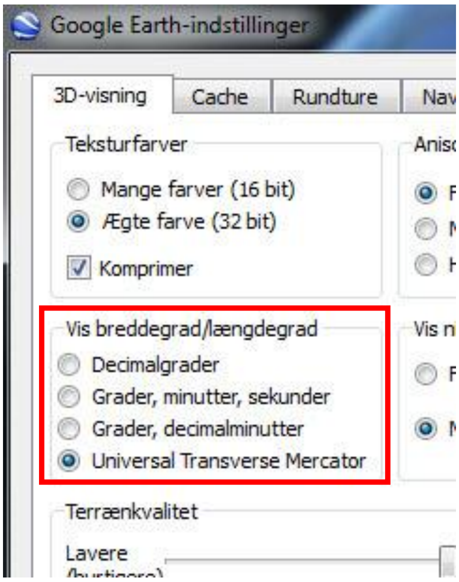


Fig. 17 In the properties menu for Google Earth you can choose between lattitude/longitude (breddegrad/længdegrad in Danish as above) and UTM (Universal Transverse Mercator)

The main point about UTM coordinates is that they locally act almost as plane Cartesian coordinates. The details on definition and theory of UTM coordinate system is omitted at this place since it will complicate things to an extent where the practical use will disappear in difficult details. See [9] (In Danish, but you can at least look at the illustrations).

The main part of Denmark is covered by UTM zone no. 32 V. It is bounded in the East-West direction by 6° and 12° meridians and center line is the 9° meridian. Northern UTM coordinate is distance from equator in meters and Eastern coordinate is measured as signed distance from center line with 500000 m added to avoid negative coordinate numbers. On the map in Fig. 18 I have shown part of zone 32 V and I have drawn a 100 km square grid cowering Denmark by simply marking waypoints at O(400000,6000000), A1(500000,6000000),...., D4(800000,6400000), etc. As you can see the further away you come from the center line the more the square grid deviates from UTM lines. In the Eastern part of Denmark you will shift to zone 33 V with center line at 15° meridian.

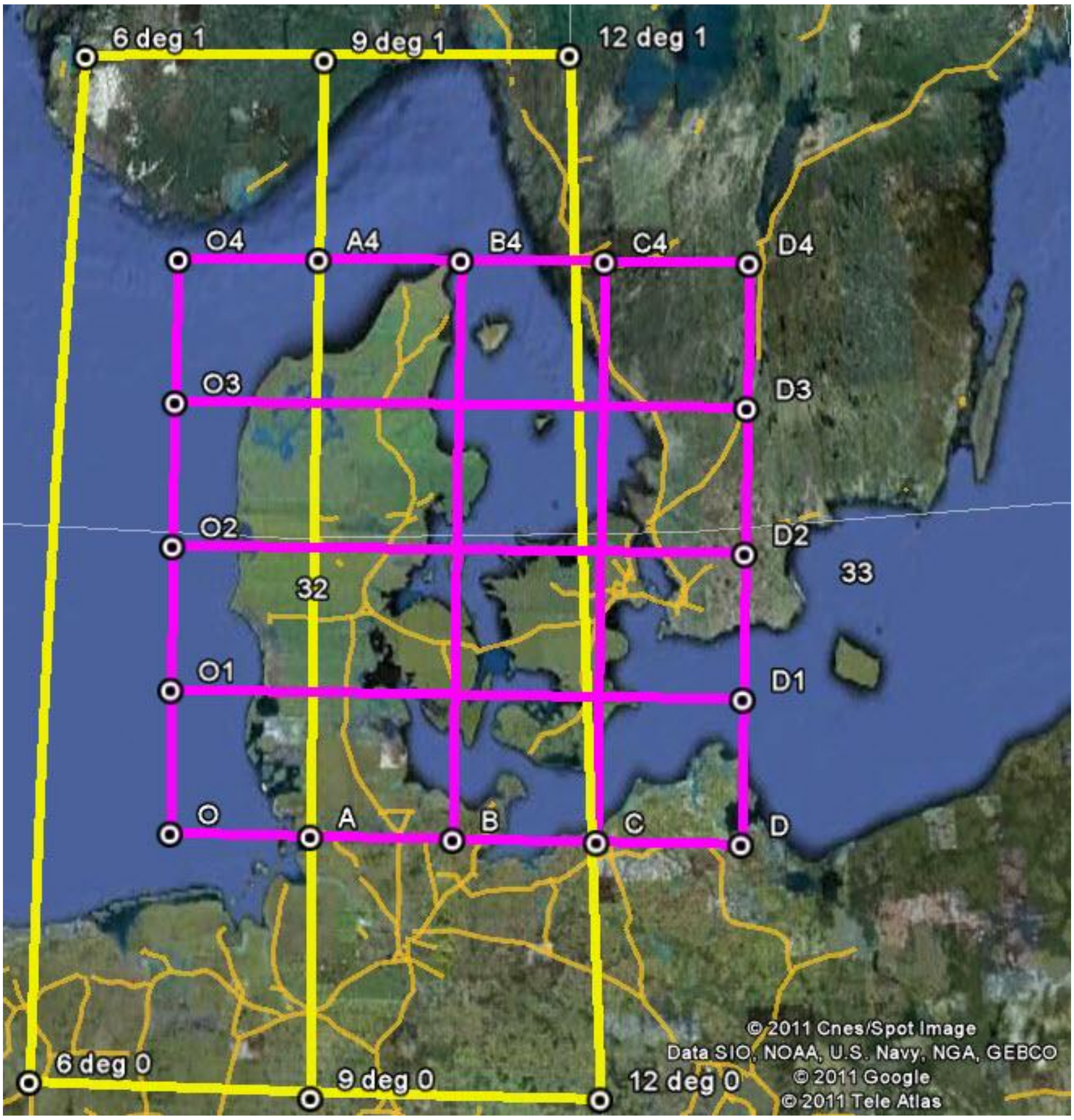


Fig. 18 Part of UTM zone 32 V which is bounded in the East-West direction by meridians 6° and 12° and has its centerline along meridian 9°. In North-South directions it extends beyond the yellow lines to lattitudes 80° South and 84° North. Also a 100 km sqare grid is marked (magenta color). Construction description for this can be read in the text above this figure.

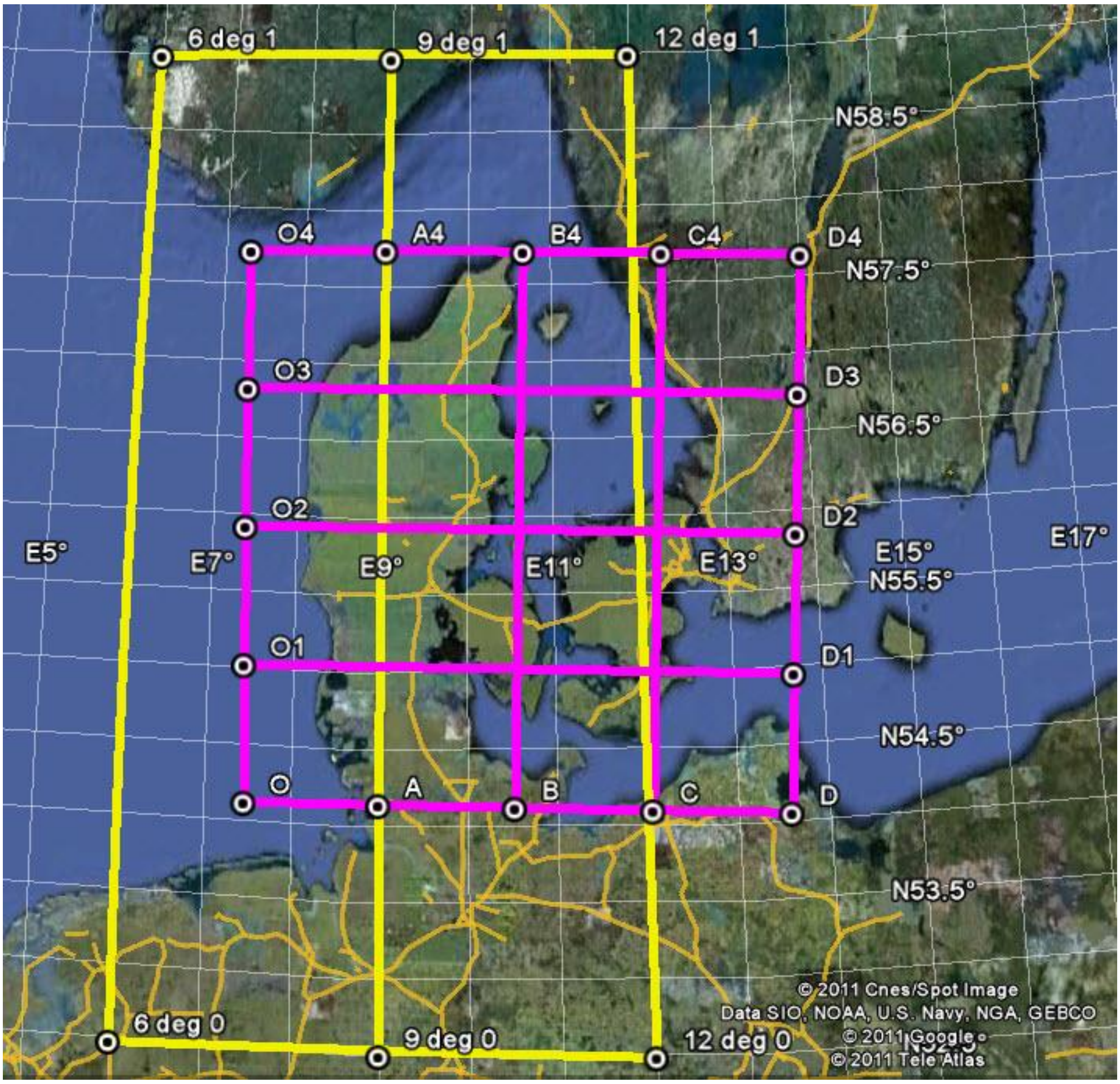


Fig. 19 The same as in Fig.18 but now with Google Earth put in lattitude/longitude mode resulting in a corresponding net of white lines.

3 Regular pentagon route constructed by using UTM coordinates

I want to plan a route shaped as a regular pentagon. A route is defined by means of a sequence of waypoints that is either made by graphical means almost as when you draw piecewise linear curves in GeoGebra. But if want a high degree of regularity you can key in the waypoints by coordinates. And if you use UTM-coordinates you can calculate the Cartesian way.

As a starting point I pick the starting point from the pentagon in Fig. 12. I use GeoGebra for the calculations but this is just one possible choice. You could choose paper, pencil, handheld calculator and usual trigonometry or you could type in formulas in a spread sheet.

I do not care about the graphics window since the size of the coordinates and the extent of the polygon makes zooming very cumbersome. I think GeoGebra needs a function for centering the graphics view at a point with a given coordinate set.

As an excercise you perhaps should do the construction with small, nice coordinate numbers first as in Fig. 20 below.

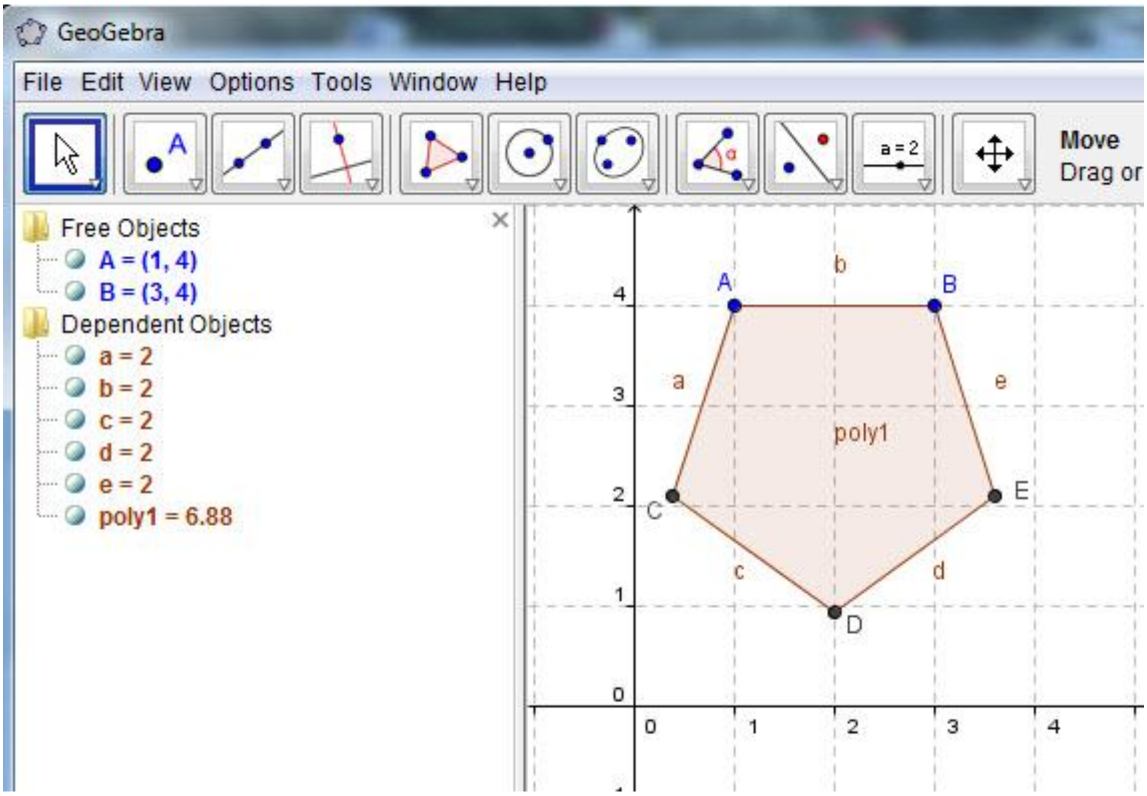
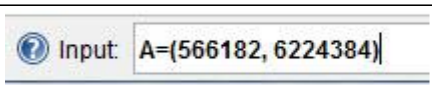
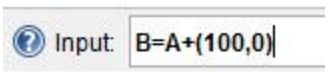
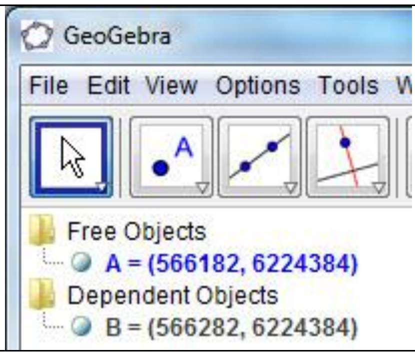


Fig. 20 You may benefit in understanding what comes next if you try it out with small, nice numbers.

	Keying in coordinates for point A in the inputline of GeoGebra
	Adding 100 m to the first coordinate (Eastern) gives B 100 m East of A
	This is what shows up in the algebra window
	Creating the pentagon by means of the built in function Polygon. Taking the points in the keyed in order makes the line AB the Northern boarder of the pentagon.

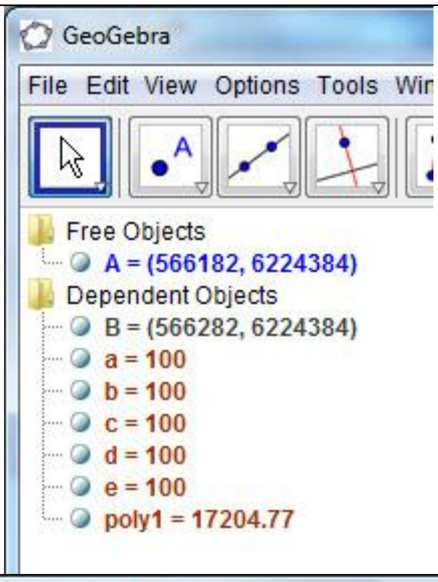
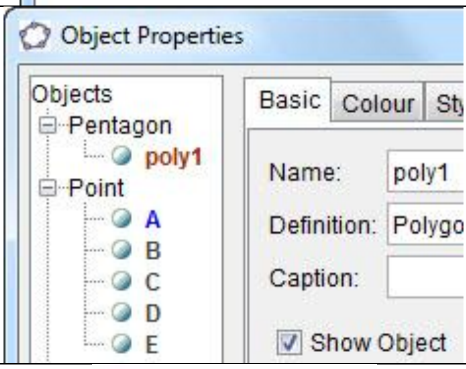
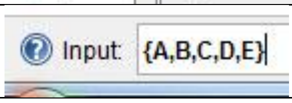
<table>
<tr><td>
</td><td>This is what is shown in the algebra window. I think it is not very informative to know the five sides are all 100 m. It is the coordinates of the vertices I need.</td></tr>
<tr><td>
</td><td>Right cliking and reading of the properties of poly1 I get to know the names of the points</td></tr>
<tr><td>
</td><td>This keying makes a list of the coordinates for the points as is seen in the algebra window below</td></tr>
</table>

Fig. 21 Calculating coordinates for the pentagon

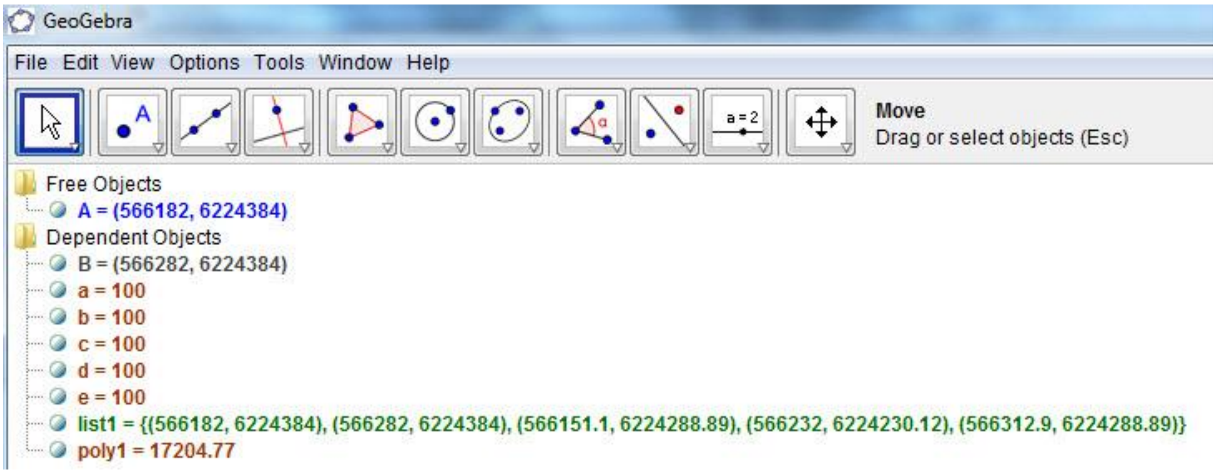


Fig. 22 Reading of coordinates for the vertices of the pentagon in the green list1

You can put your GPS in UTM mode and enter the coordinates directly creating waypoints but I find it easier to do this part by the software coming with the GPS.

Fig. 23 In the MapSource menu for creating new waypoint you enter the desired name and the coordinate read of (Fig. 22)

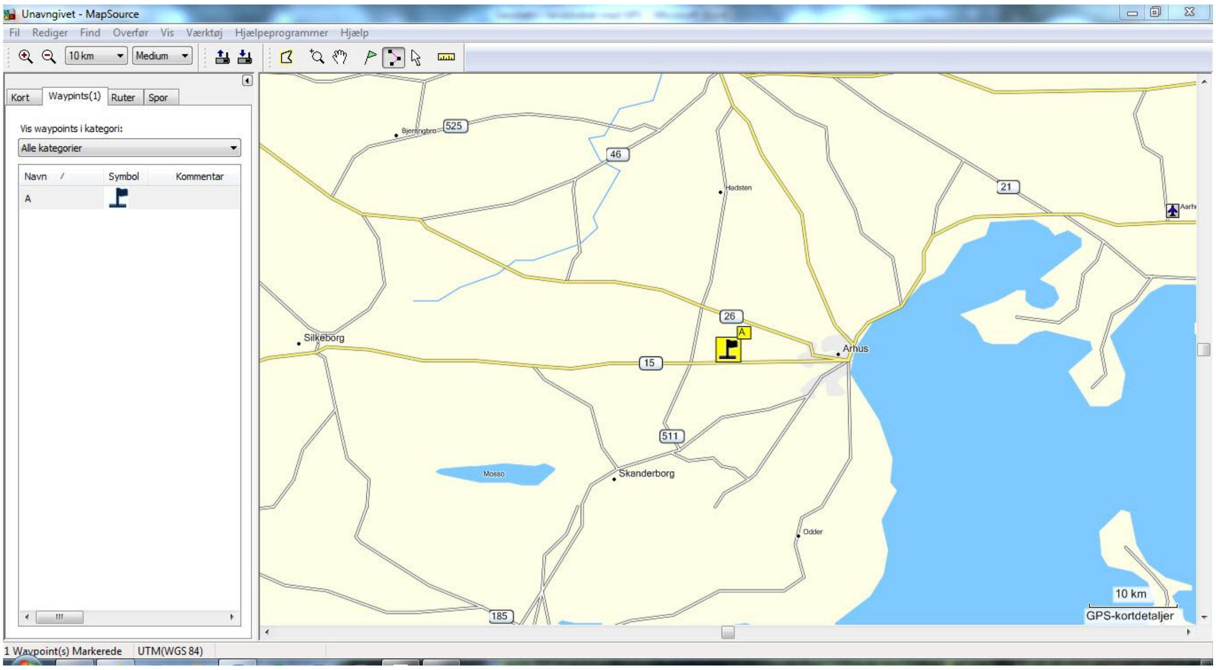


Fig. 24 Waypoint appears in the MapSource map window

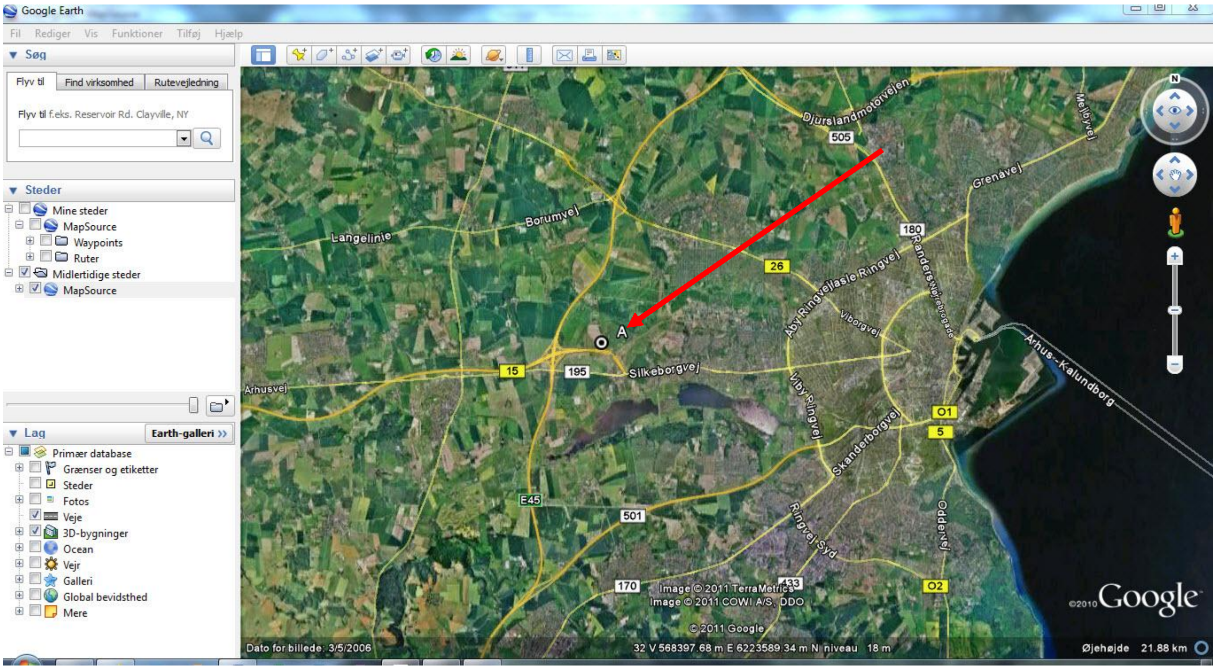


Fig. 25 Switching to Google Earth also shows the waypoint A.

Having entered the remaining four vertices as waypoints you can create a route by another menu

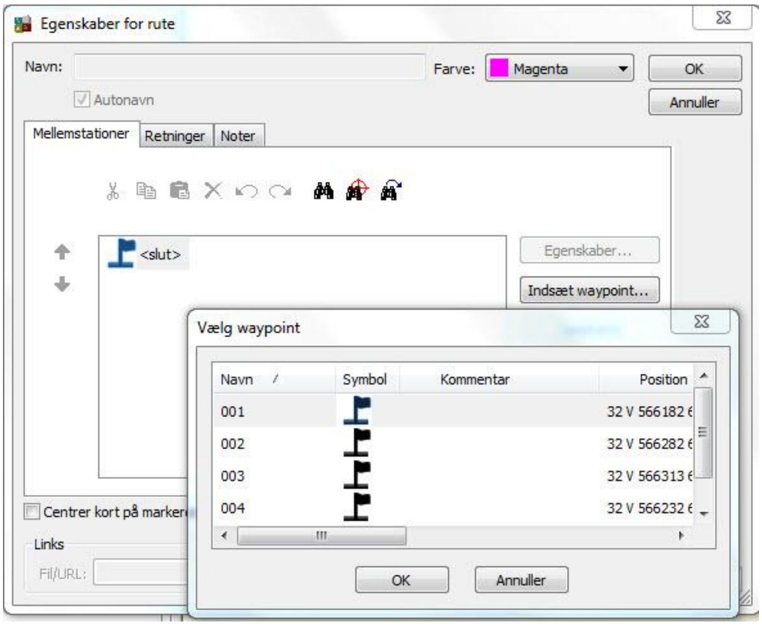


Fig. 26 Creating a route from waypoints

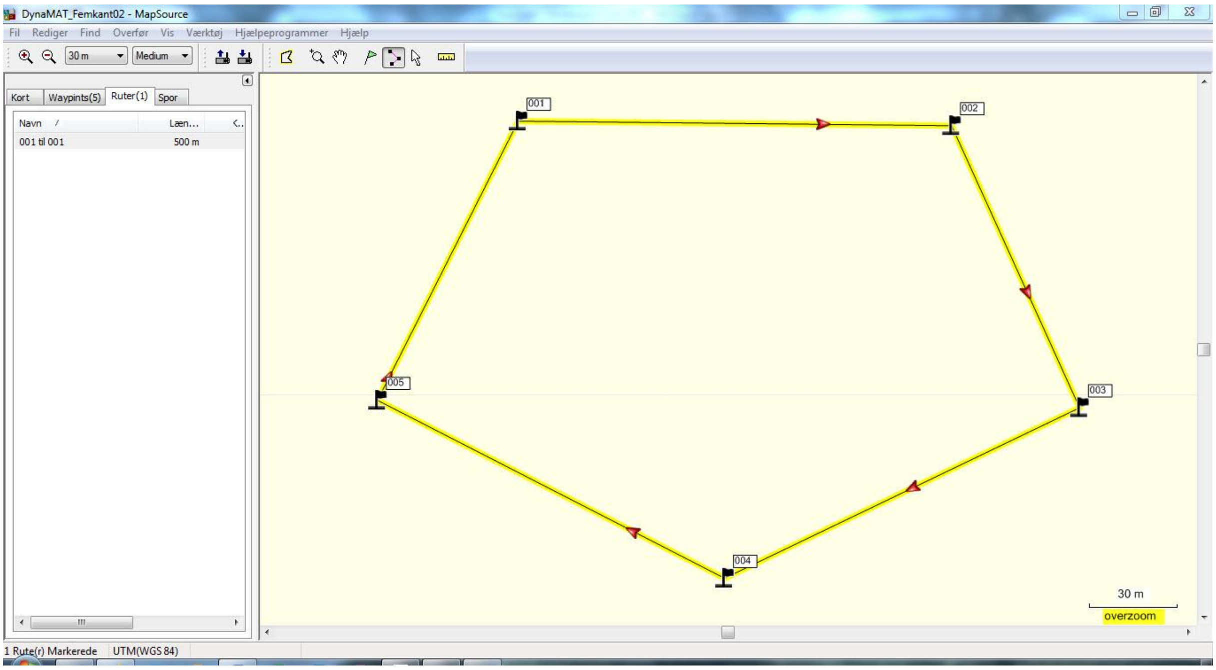


Fig. 27 The created route shows up in the map window

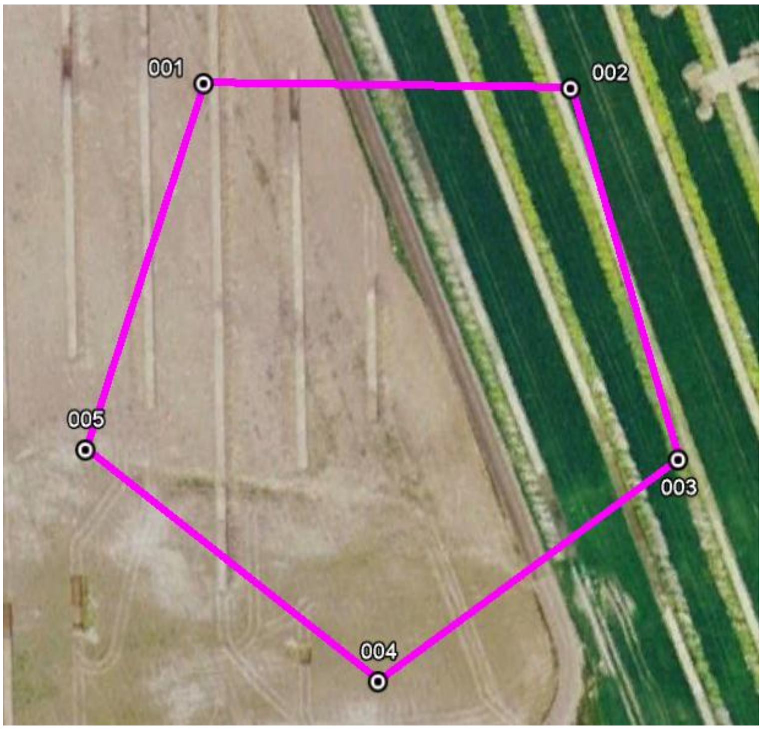


Fig. 28 Uploading the route to Google Earth produces this picture

If one expects to be able to walk such a regular pentagon in the fields disappointment lies right ahead. You learn, experience and reflect about inaccuracies and their sources. This can be imperfections in GPS navigation. Although the calculated deviation can be down to few meters under good conditions there can occur situations with much larger errors e.g. due to poor satellite coverage. This is seen in the next picture Fig. 29 showing parts of two different jogging trips measured with the same GPS device along the same route in Vienna.

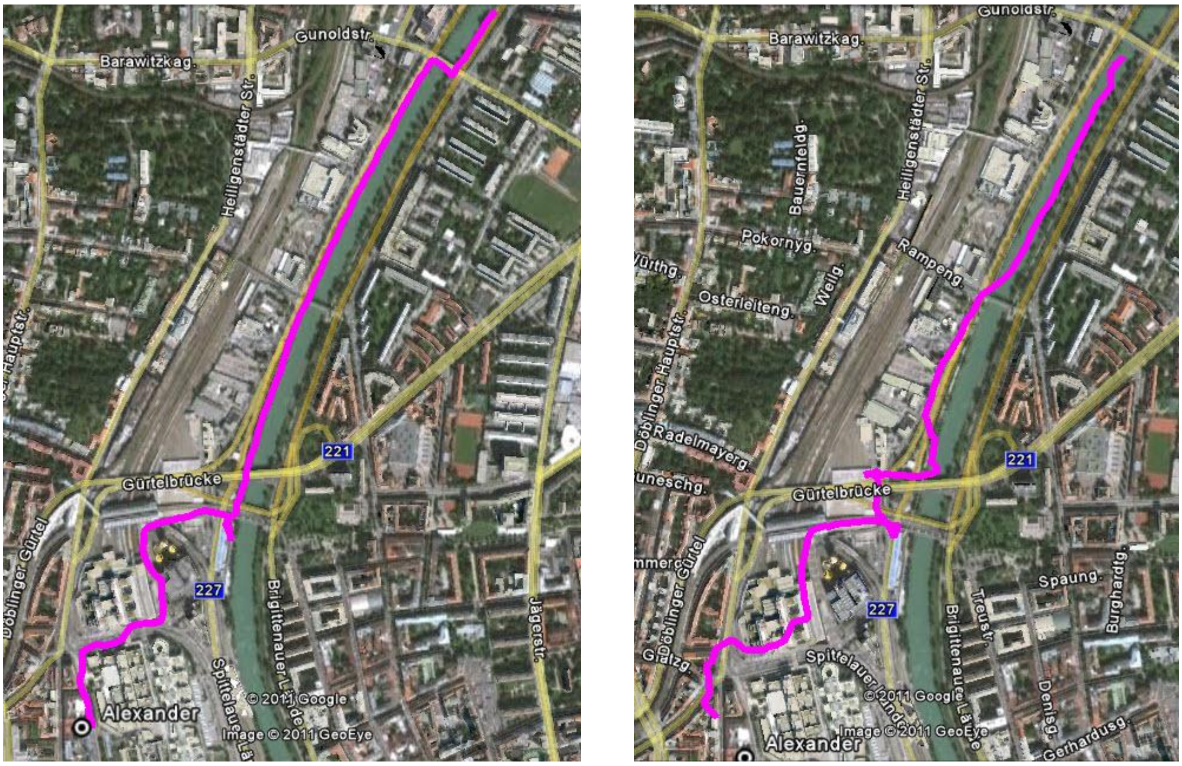


Fig. 29 Same physical route along Donaukanal in Vienna, same GPS device used, two different trips 28th and 29th of April 2011

Another source of imperfection is difficulties in reading of directions from GPS display and follow them exact once you are in the fields with no constructed roads to follow. Below you see resulting tracks from actually following GPS directions in the landscape.

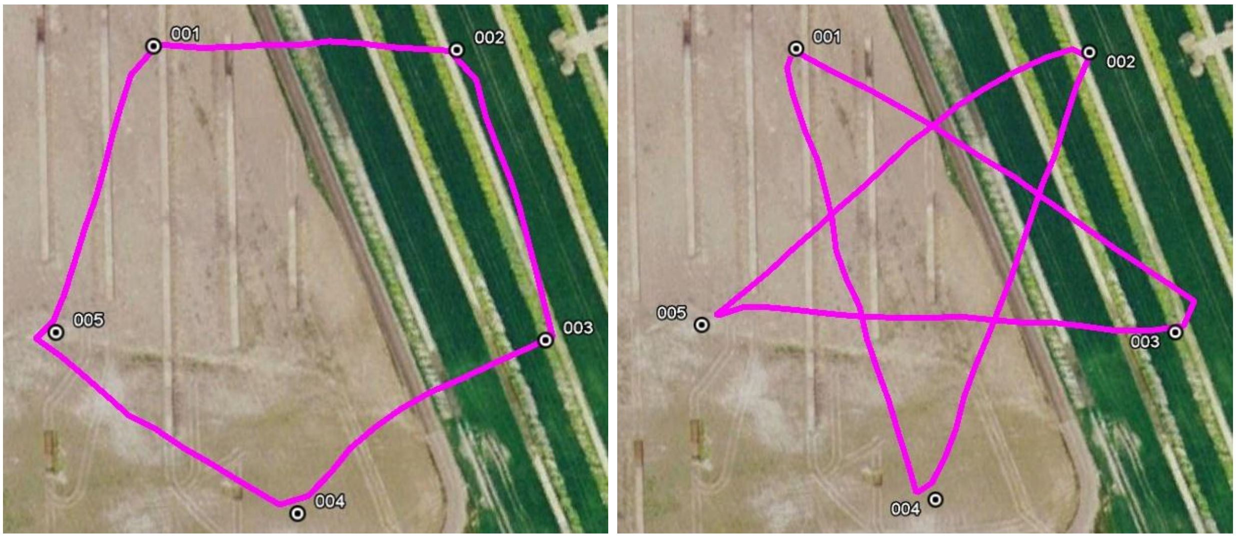


Fig. 30 Regular pentagon and pentagramme trace from waypoint to waypoint.

4 From parametric equations to routes in the landscape

In this chapter I walk the plank and transfer a curve given by parametric equations to a route in the landscape. As an example I decide to walk a flower

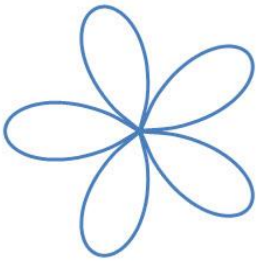


Fig. 31 The flower I decide to walk

Coordinates for the curve in Fig. 31 are calculated from parametric equations using spreadsheet (Excel 2007).

	A	B	C	D	E	F	G	H
1	t	x_UTM	y_UTM	dt	r	n	x0_UTM	y0_UTM
2	0	566232	6224315	3	100	5	566232	6224315
3	3	566257,7	6224318			k		
4	6	566280,9	6224325			2		
5	9	566299,2	6224337					
6	12	566311,1	6224350					
7	15	566315,7	6224363					
8	18	566312,9	6224374					
9	21	566303,8	6224380					
10	24	566289,9	6224379					
11	27	566273,6	6224372					
12	30	566257	6224358					
13	33	566242,5	6224339					
14	36	566232	6224315					
15	39	566237,4	6224340					
16	42	566237,2	6224365					
17	45	566232	6224386					
18	48	566222,9	6224401					
19	51	566211,9	6224409					
20	54	566201,1	6224410					
21	57	566192,7	6224403					
22	60	566188,7	6224390					
23	63	566190,4	6224372					
24	66	566198,5	6224352					
25	69	566212,8	6224332					
26	72	566232	6224315					
27	75	566209,6	6224328					
28	78	566186,3	6224335					

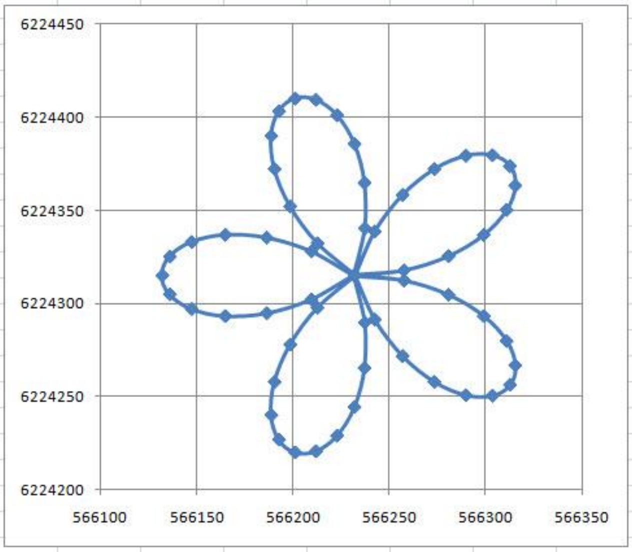


Fig. 32 Screen shot from the calculations of the flower

The formulas for the flower are entered in cells B2 and C2 and then copied downwards. Be careful to take notice of the use of absolute (marked with $) and relative cell references. Formulas are shown in the following two lines

```
x-coordinates: =$E$2*ABS(SIN($F$2*RADIANER(A2)))*COS($F$4*RADIANER(A2))+$G$2
y-coordinates: =$E$2*ABS(SIN($F$2*RADIANER(A2)))*SIN($F$4*RADIANER(A2))+$H$2
```

Here follows a brief description of the structure of the spreadsheet

Column A: Degrees from 0° and up until sufficiently many points are calculated

Column B: x-coordinates for the curve

Column C: y-coordinates for the curve

Column D: Steps for degrees in column A

Column E: Radius for circumsscribed circle

Column F: n = number of petals, k factor for scaling degrees from column A

Columns G and H: UTM coordinates for center of flower

Next step is to enter the coordinates from column B and C as coordinates for waypoints in your GPS software. But this should not be a task for manual keying in, so what to do? It is unfortunately not possible to copy coordinates directely from Excel into MapSource.

Searcing the internet I found a freeware program GPS Babel [10] that makes it possible to make a lot of conversions. Unfortunately I was unable to set it to transform directely from my Excel file UTM coordinates to MapSourse. A little detour brought me the data I wanted.

First of all I had to transform my UTM coordinates into lattitude/longitude coordinates. (This may sound very awkward if GPS Babel can't transform UTM coordinates directely and I think it can but until now I have not been able to make i work).

Once again searching the internet I found lots of sources that can do the transformation one point at a time. Bigger programmes seemed to be expensive. But at source [6] I found an Excel Spreadssheet that solves the problem. I only had to copy in my own coordinate columns and copy formulas sufficiently many rows down.

I have checked to program with a number of points where I knew coordinates both in UTM and in lattitude/ longitude mode. It seems to be alright. I will not go deeper into the formulas behind. That must be left for another project.

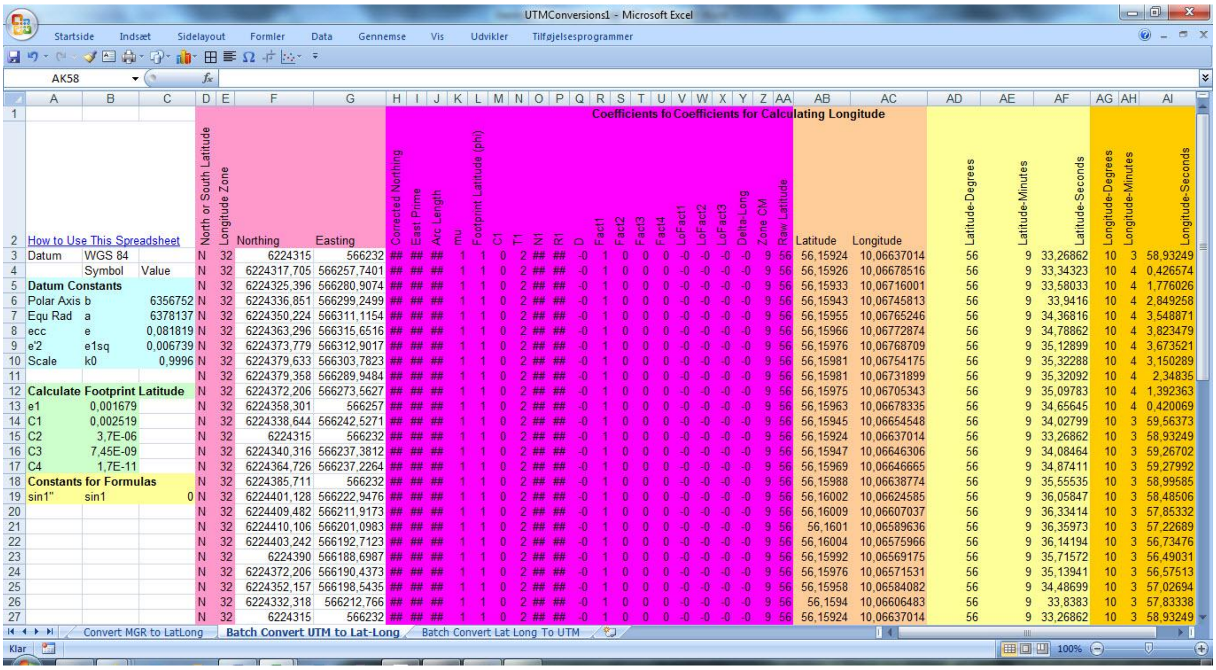


Fig. 33 Screen shot from Excel spreadsheet for transforming from UTM coordinates to lattitude/longitud coordinates [6]. Please zoom to see details.

Copying the coordinates from column AB and AC into a texfile replacing decimal kommas (Danish standard) with decimalpoints (international standard) an inserting kommas between koordinates (all by search and replace function) puts the datas into a form that GPS Babel will eat.

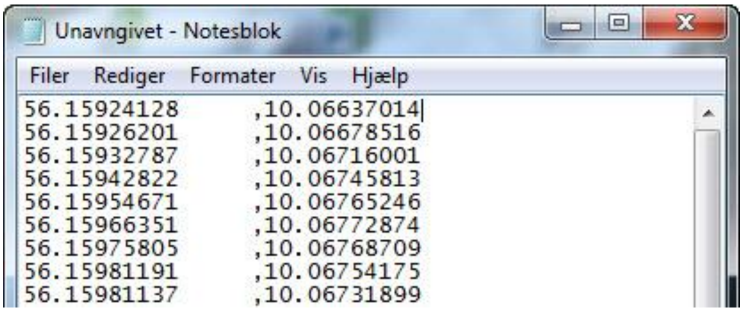


Fig. 34 Comma seperated text file with coordinates ready to be entered into GPS Babel

The GPS Babel transformation to a MapSource file can now take place

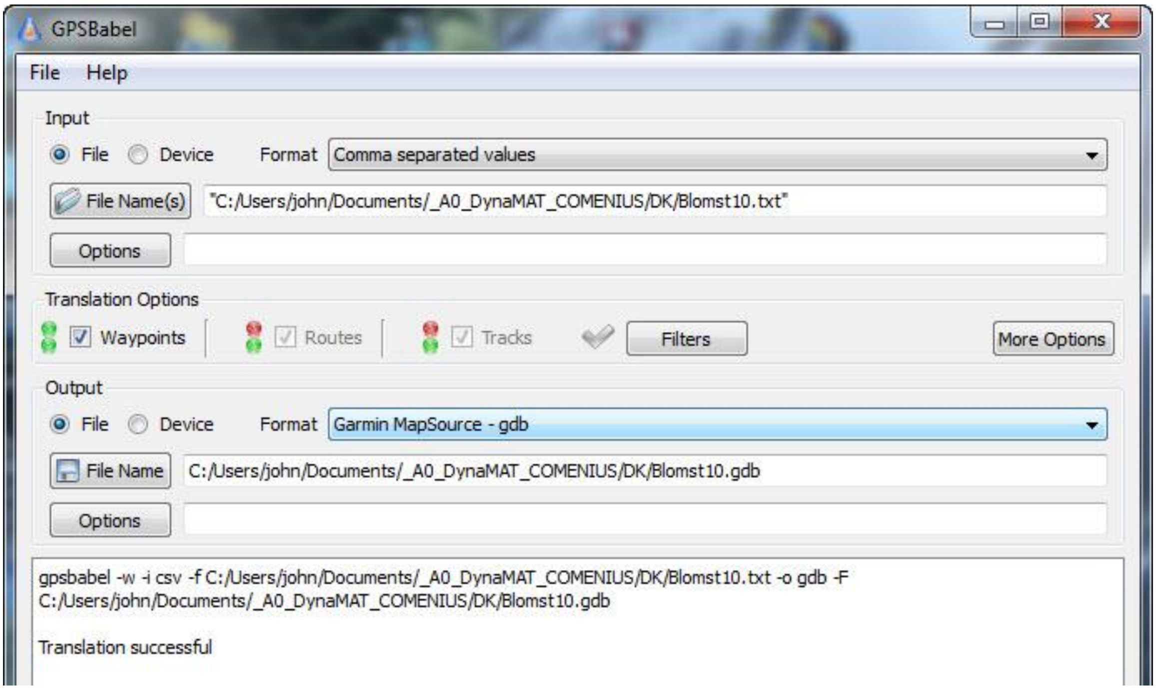


Fig. 35 Screenshot from GPS Babel. Translation successful. YES! ☺

Next step is to open the resulting MapSource file in MapSource to see what the result looks like here. And as you see in Fig. 36 it seems that the project is moving towards a succesful outcome

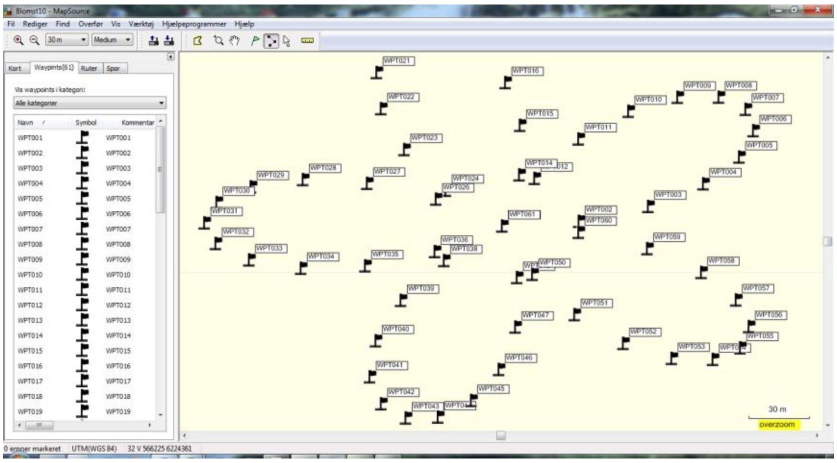


Fig. 36 Screen shot from MapSource showing waypoints created from Excel generated coordinates.

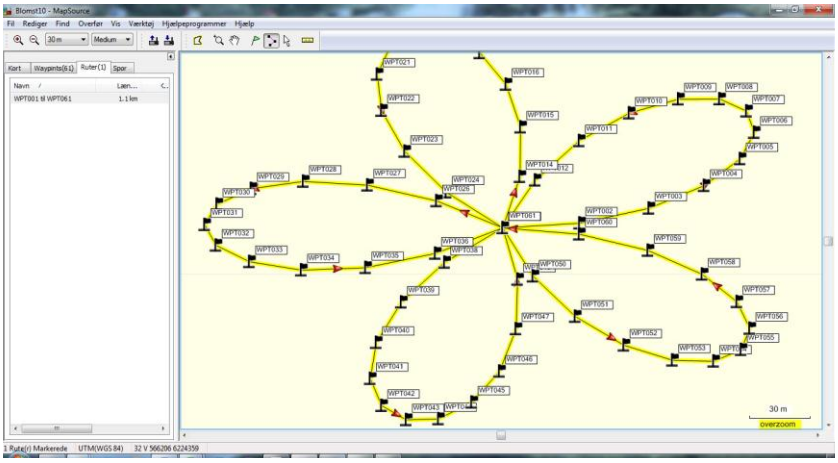


Fig. 37 Route created from waypoints in Fig. 36

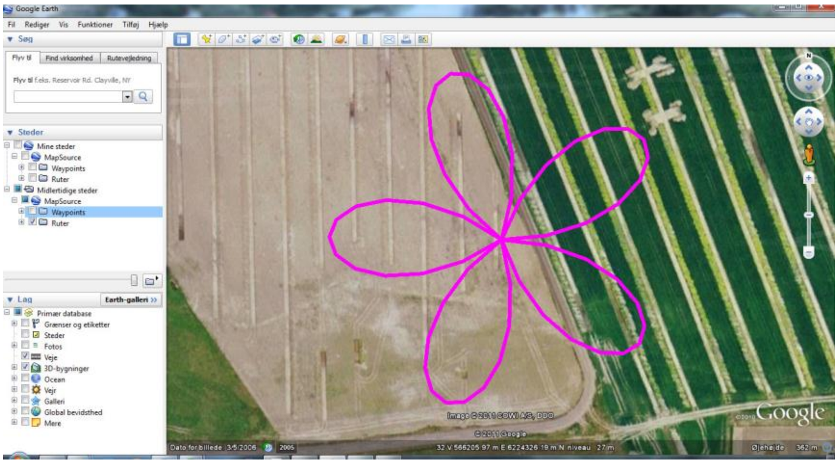


Fig. 38 Flower route transferred to Google Earth

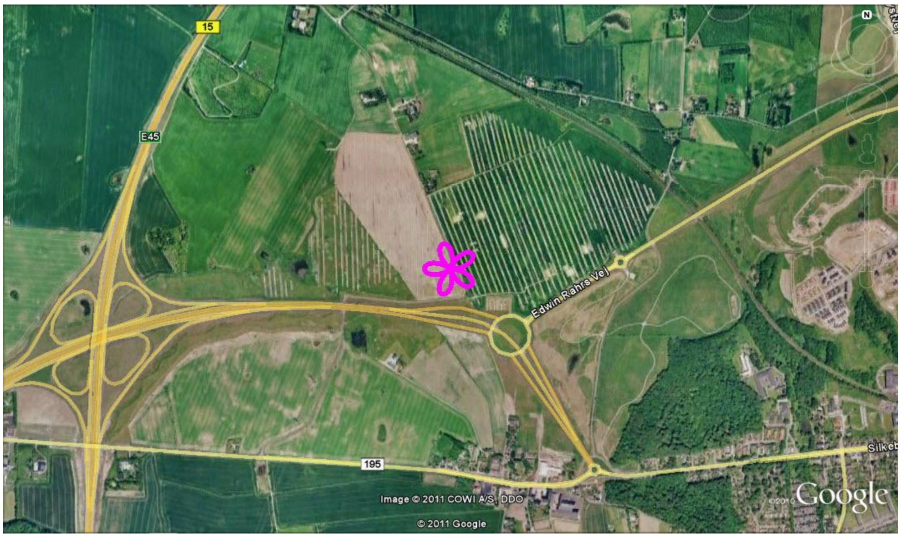


Fig. 39 From a more elevated point of wiev it seems as if the flower can be a rival to the nearby cloverleaf highway interchange.

Now lets try to mix the pentagon from section 3 with the flower. The picture turns out like Fig. 40

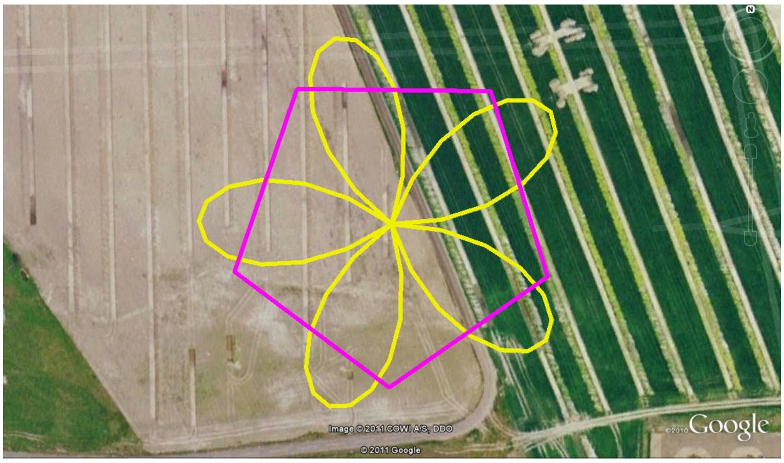


Fig. 40 Pentagon from section 3 and flower in same Google Eart picture

Someone may think why not adjust the flower to the pentagon. At least I think so when looking at Fig. 39. But working with a dynamical attitude and dynamical tools nothing can be more natural than to go back to the spreadsheet, change a few parameters and end up with the route seen i Fig. 41

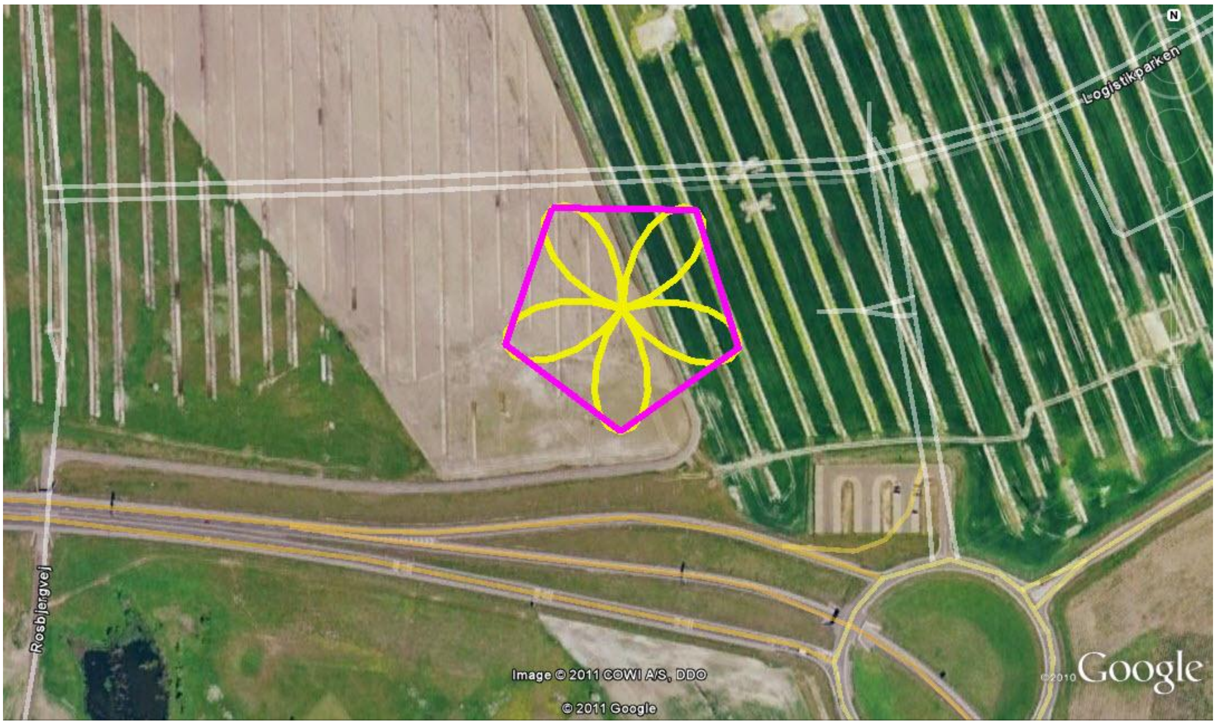


Fig. 41 Vertices of flower's petals are now concurrent with vertices of pentagon

Finally it is time to go out into the field to test calculations against practical GPS navigation. Therefore I transfer the flowerroute to the GPS devise. The resulting track (magenta) is shown i Fig. 42 including a comparison to the computergenerated yellow path.

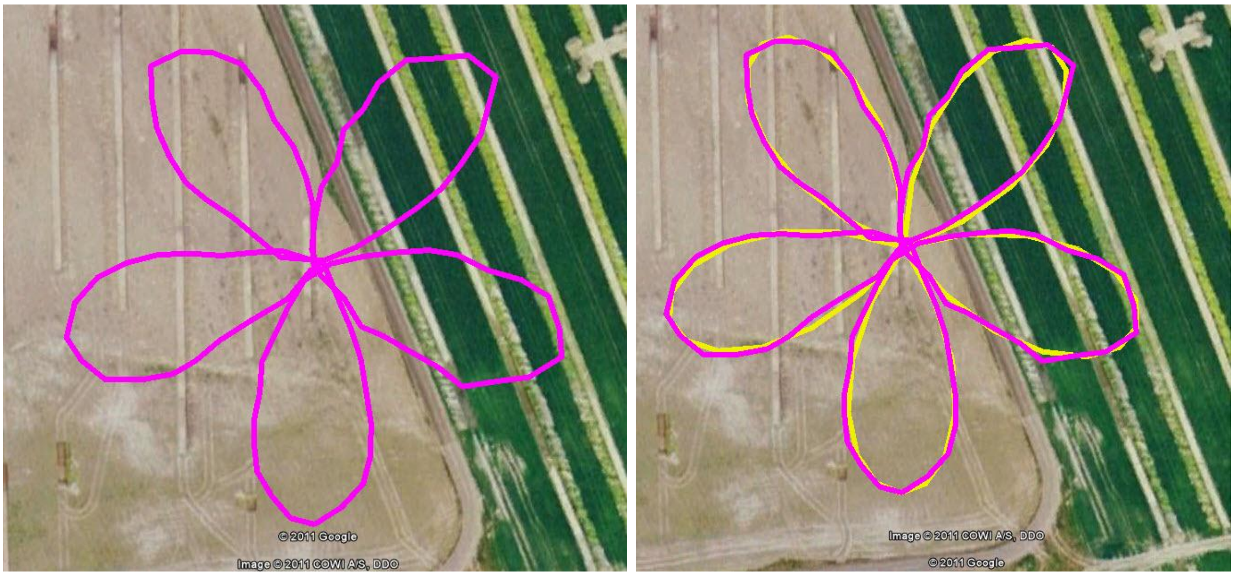


Fig. 42 Following fhe flower route in the landscape, tracking the route and comparing ideal track (yellow) with walked track (magenta)

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