From static to dynamical problem posing.

Vladimir Georgiev, Yuki Kurokawa

1 Short introduction

During the activities of the problem posing labs in Pisa it was interesting to know something about the tools and methods used in other countries to introduce some math arguments in more attractive way. This possibility occurs, since Prof. Kurokawa was a guest of the Department of Mathematics in the period april - september 2011.

The main goal of this section is the following:

- to see how the work in problem posing lab in Pisa can be combined with the Japanese experience in math teaching.

One of the problem posing labs working in the Spring of 2011 at Pisa had international participation: students (future teachers) from Spain and Italy. The work of the lab was organized by the authors of this section, i.e. we had international participation at any level.

2 Examples from different levels

We start with very elementary examples of famous Japanese problems with calculation without use of any equation. It is important to underline that the origin of these problems and examples is not artificial and come from real life problems.

1. (Cranes and Turtles Calculation) The example is based on a ”real life” situation (see Figure 1) and for this we need initially a nice model.

Figure 1: Cranes and Turtles Calculation

The model is based on the following observation (see Figure 2):

If we have 70 heads (cranes or turtles) then the number of the legs depends how many are the cranes. The possibilities are:

- we have 70 cranes and 0 turtles;
- we have 69 cranes and 1 turtle;
- we have 68 cranes and 2 turtles;

- ⋯

These possibilities are given in the Table 1.

Heads (70)				
Cranes	70	69	68	⋯
Turtles	0	1	2	⋯

(1)

Now we turn to a calculation of the legs. This is done in the Table

Legs (222)				
Cranes	140	138	136	⋯
Turtles	0	4	4	⋯
TOTAL	140	142	144	⋯

(2)

It is clear that the last row in this table starts from 140 and each next term is obtained adding 2, i.e.

$$140(+2) \Rightarrow 142(+2) \Rightarrow 144(+2) \Rightarrow \cdots$$

The question now is:

- How many steps we have to do in order to start from 140 and arrive to 222?

For example if we have to start from 140 and arrive at 142 we have to make

$$\frac{142 - 140}{2} = 1$$

step, to start from 140 and arrive to 144 we need

$$\frac{144 - 140}{2} = 2$$

steps and to start from 140 and arrive at 222 we need

$$\frac{222 - 140}{2} = 41$$

steps. Since any step requires to diminish the number of cranes by one, we shall have

$$70 - 41 = 29$$

cranes and 41 turtles.

Discussions about this problem were concentrated on the fact that the problem can be seen from other point of view as equation

$$4x + 2y = 222,$$

where x is the number of turtles and y is the number of cranes. The condition that the total number of cranes and turtles is 70 can be expressed as

$$x + y = 70.$$

This point of view is not working when primary school pupils are involved.

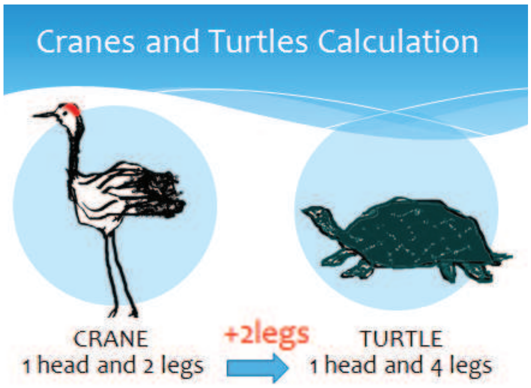

Figure 2: Model for Cranes and Turtles Calculation

Possible further problems are well - known as problems for cranes, turtles and dragonflies and in this problem not only the legs, but also the winds are taken into account (see Figure 3). However, using tables similar to Tables 1 and 2, we can find a solution to this problem.

Hint. Dragonflies are 9, total number of cranes and turtles is 35, and the total number of legs of cranes and turtles is 118. One can apply the solution of the previous problem to find the number of cranes and turtles separately.

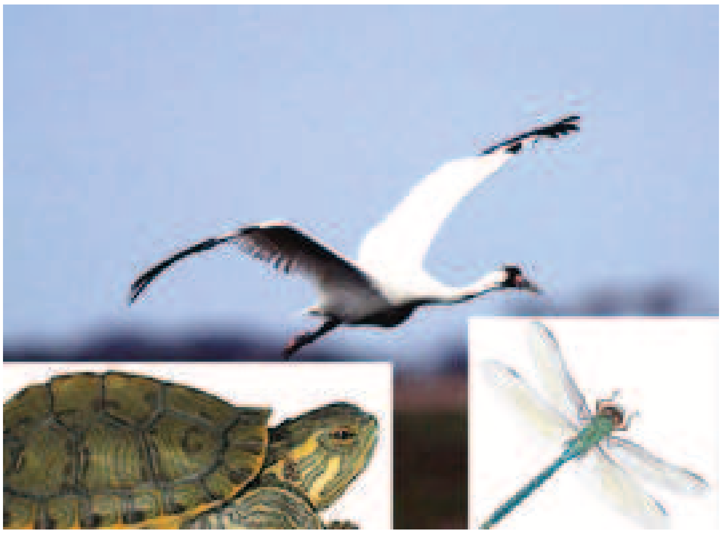

Figure 3: Problem for Cranes, Turtles and Dragonflies

It is interesting to compare the Japanese experience with the experience of the group math competitions organized by the Department of Mathematics, Pisa University. One example:

Exercise 1. *A bat ate 1050 dragonflies on four consecutive nights. Each night she ate 25 more than on the night before. How many did she eat each night? Solve this algebraically.*

It is clear that this problem can be solved easily denoting by x the number of dragonflies that the bat has eaten in the first night. Then we have the simple equation:

$$x + (x + 25) + (x + 50) + (x + 75) = 1050$$

Figure 4: Problem for Bats and Dragonflies

so $4x + 150 = 1050$ and we find $x = 225$.

During the work of the Math Lab in Pisa a specific game with Flash script was compared with the above examples. The game is connected with the following Diophantine equation

$$Bx = A + Cy, \tag{3}$$

where A, B, C are natural numbers that we choose before starting the game, while x, y can be interpreted by two buttons (the green on Figure 5 and the red one on the same Figure).

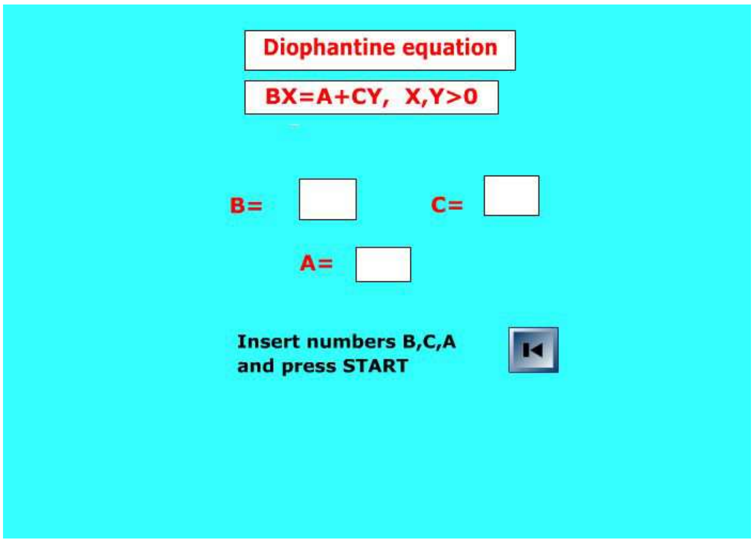

Figure 5: Game with Diophantine equation

If one presses the green button this corresponds to the operation

$$x \Rightarrow x + 1,$$

or

$$Bx \Rightarrow Bx + B.$$

If the red button is pressed, then the following action

$$y \Rightarrow y + 1,$$

or

$$Cy \Rightarrow Cy + C$$

is executed. For this once the prof introduces the numbers B, C, A, the student (player) chooses his strategy: pressing $x --$ times the green button and $y --$ times the red one, the program computes $Bx - Cy$ and if this number coincides with A then the exit is successful and the number of all actions

$$x + y$$

is presented on the screen.

The *score* is exactly this number $x + y$ and the purpose of the game with many players is to have minimal sum $x + y$. One example is the choice $B = 3, C = 7, A = 1$ chosen by the prof and $x = 12, y = 5$ with "score" $12 + 5 = 17$ is on the Figure 6.

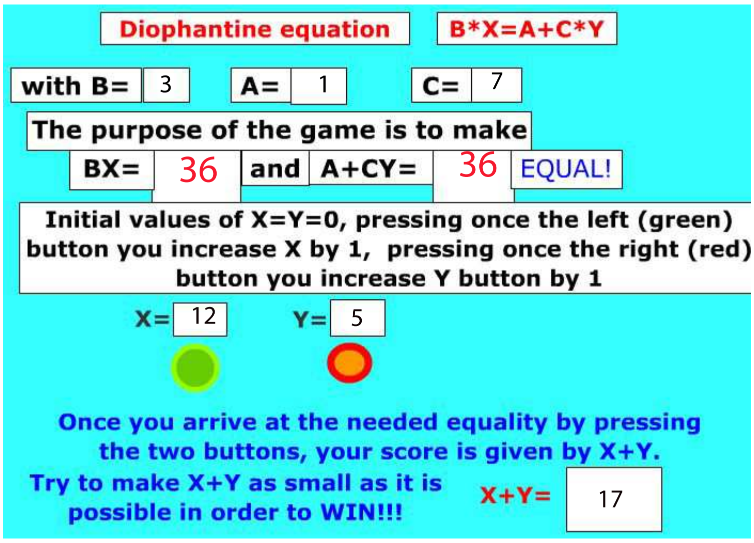

Figure 6: Game with Diophantine equation

One can find a better solution $x = 5, y = 2$ with a better score $5 + 2 = 7$.

The following link enables one to play this game

Link to the game Diophantine Game

There are several questions that have been discussed in the lab:

- find triples A, B, C such that the Diophantine equation has no solution;
- find a necessary and sufficient conditions so the the Diophantine equation has at least one solution;
- find an algorithm to win or to find optimal $x + y$.

The results of this attempt to combine the Italian and Japanese teaching experience are very promising and show the possibilities for further developments.

3 Tasks for further development

Possible argument for further treatment is presented on Figure 7.

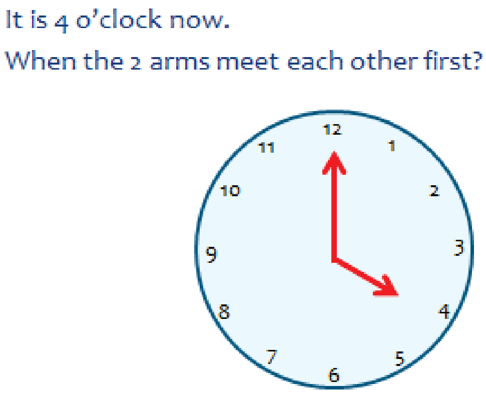

Figure 7: Clock problem

Possible discussions and solution can be found by using the following.

Hint. Each minute each of the arms makes a rotation in clockwise direction of

$$Longarm : 360^0/60 = 6^0,$$

$$Shortarm : 30^0/60 = 0.5^0.$$

Thus, each minute the angle between two arms will diminish by

$$6^0 - 0.5^0 = 5.5^0.$$

The angle between two arms at 4 oclock is 120^0. Therefore,

$$120/5.5 = 21.81818181^0.$$

Answer.

$$4 : 21 : 49.$$

Another argument introduced and discussed in the problem posing lab is presented on Figure 8.

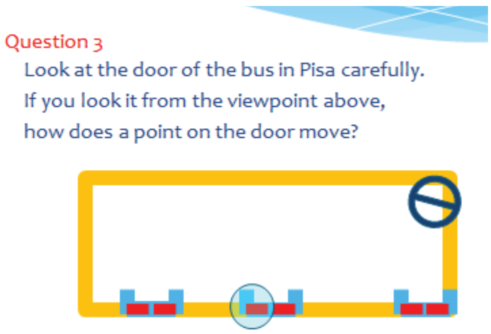

Figure 8: Door problem

Formally, participants of the lab had the following suggestion: try to look around you and introduce some math model and problem. One of the most frequently used vehicles in Pisa are buses of an old generation having a system to open the door presented on Figure 8. In some sense the problem can be connected with one of the several examples that have been presented to the participants of the lab. Among these examples we had the following problem taken from Chapter "Astronomy and instruments to draw quadratic curves" in [2].

Exercise 2. *(see chapter "Astronomy and instruments to draw quadratic curves" in [2]). A student of the famous Galileo Galilei discovered a new planet, which orbits the Sun on an elliptic orbit with semi-axes a and b. Suppose that an observer is located at a point in the plane of the ellipse, such that the ellipse is seen from that point at an angle of* 90^0*. Compute the distance between the center of the ellipse and the observation point.*

One can use Cartesian coordinates and find an analytic solution describing by an equation the set of points such that the ellipse is seen from that point at a right angle. Problems of this type can be seen in different places. One can see for example in [4] for some other attractive examples connected with conics. Since the practical experience of pupils in High School is not at very high level, we insisted in the problem posing lab to continue the line from [1], [2]. One can use GeoGebra for the problems (2). However, Japanese colleagues that have no information about the existence of GeoGebra tried another approach based on everyday IT tools used in their classroom teaching. The Power Point application is a real animation prepared with Microsoft Office 2010 the can be seen from any free power point viewers.

Here is the link to the ppt file Power Point application Disadvantage is the fact that one needs Power Point (even older versions) that is not open source software.

The next step was to treat the Door Problem described on Figure 8. It is interesting that the first approach of the Japanese colleagues was based on the Power Point animation.

However, the first of the tasks to use GeoGebra applications was done exactly for the Door problems

The link to the Geogebra file is here

This example enables one to compare the capabilities of Geogebra and Power Point and see the GeoGebra is one tool adapted to Math teaching.

Other possible tasks are presented by the student Eva Cricca in the Power Point presentation and the Geogebra applications

- optimize the distance between two points, when the obstacle is a river, one can see the The link to the Power Point file is here
- optimize the distance between two points when the obstacle i a lake (ellipse). and the The link to the Geogebra file is here

References

[1] Georgiev, V., Mushkarov, O., Ulovec, A., Dimitrova, N., Mogensen, A., Sendova, E. MEETING in Mathematics, Demetra Publishing House, Sofia, 2008

[2] A. Ulovec, J.Anderson, S. Čeretková, N. Dimitrova, V. Georgiev, O.Mushkarov E Sendova, *MATH to EARTH* 2010.

[3] Hiroku Endou , *Algorithmic girl,* 2006, Japan.

[4] J. Šunderlk and E. Barcková, *Best spot - investigation with circles*, chapter in this book.

[5] M. Yoshida (revised and commented by S. Ohya) ”Jinkoki”, Iwanami Shoten, Japan, 1977.