

Dyna MAT

From static to dynamical problem posing. Modelling optical lenses with Dynamic Geometry Software Andreas Ulovec

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1 Short introduction

During the activities of the problem posing labs in Pisa it was interesting to know something about the tools and methods used in other countries to introduce some math arguments in more attractive way. This possibility occurs, since Prof. Kurokawa was a guest of the Department of Mathematics in the period april - september 2011. During the activities of the problem posing fabs in 1 is a low superior solution.

The main goal of this section is the following: the main goal of this section is the following.

 \bullet to see how the work in problem posing lab in Pisa can be combined with the Japanese experience in math teaching.

One of the problem posing labs working in the Spring of 2011 at Pisa had international participation: students (future teachers) from Spain and Italy. The work of the lab was organized by parton: statents (tatale teachers) from spain and ratify. The work of the mas was organized the authors of this section, i.e. we had international participation at any level. lie authors of this section, i.e. we had international participation at any lever.

2 Examples from different levels ϵ – Laampies from unierent levels – again mathematicale to calculate the angle in a

We start with very elementary examples of famous Japanese problems with calculation without use of any equation. It is important to underline that the origin of these problems and examples near the centre of the lenses and the lenses and the centre of the centre of the centre, the calculations and come from real life problems. become more complex and from the extens, alone it would be different to see what happens.

1. (Cranes and Turtles Calculation) The example is based on a "real life" situation (see Figure [1\)](#page-0-0) and for this we need initially a nice model.

Figure 1: Cranes and Turtles Calculation

The model is based on the following observation (see Figure [2\)](#page-2-0):

If we have 70 heads (cranes or turtles) then the number of the legs depends how many are the cranes. The possibilities are:

- • we have 70 cranes and 0 turtles;
- \bullet we have 69 cranes and 1 turtle;
- \bullet we have 68 cranes and 2 turtles;

Dyna MAT

• · · ·

These possibilities are given in the Table [1.](#page-1-0)

Now we turn to a calculation of the legs. This is done in the Table done we cann go a calculation of the registration provided in the Table

It is clear that the last row in this table starts from 140 and each next term is obtained adding 2, i.e. for mathematics teachers. We have in the mathematics? There is a lot of it is a lot of it in the mathematics? If a ray of it is a r

$$
140(+2) \Rightarrow 142(+2) \Rightarrow 144(+2) \Rightarrow \cdots
$$

The question now is:

 \bullet How many steps we have to do in order to start from 140 and arrive to 222? \mathcal{L} the centre of the lenses and light being more off-centre, the calculations and light being more of \mathcal{L}

For example if we have to start from 140 and arrive at 142 we have to make \overline{O} calliption we have to start from 140 and arrive at 142 we have to make

$$
\frac{142-140}{2} = 1
$$

step, to start from 140 and arrive to 144 we need **2.1 Reflection**

$$
\frac{144-140}{2} = 2
$$

steps and to start from 140 and arrive at 222 we need

$$
\frac{222 - 140}{2} = 41
$$

steps. Since any step requires to diminish the number of cranes by one, we shall have

$$
70-41=29
$$

cranes and 41 turtles.

Discussions about this problem were concentrated on the fact that the problem can be seen from other point of view as equation

$$
4x + 2y = 222,
$$

where x is the number of turtles and y is the number of cranes. The condition that the total number of cranes and turtles is 70 can be expressed as

$$
x + y = 70.
$$

This point of view is not working when primary school pupils are involved.

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Figure 2: Model for Cranes and Turtles Calculation 1 gare 2. Hough for Grands and Tartitos of

Possible further problems are well - known as problems for cranes, turtles and dragonflies and in this problem not only the legs, but also the winds are taken into account (see Figure [3\)](#page-2-1). However, using tables similar to Tables [1](#page-1-0) and [2,](#page-1-1) we can find a solution to this problem. T material can be useful for science teachers, who can use it to model experiments with lenses, with l reflection and reflection and α is a problem for cranes, turties and dragonities ϵ

Hint. Dragonflies are 9, total number of cranes and turtles is 35, and the total number of legs of cranes and turtles is 118 . One can apply the solution of the previous problem to find the number of cranes and turtles separately. and proposition performance of glass and continues the glass and the same happens when the same happens when the same $\frac{1}{2}$

Figure 3: Problem for Cranes, Turtles and Dragonflies

It is interesting to compare the Japanese experience with the experience of the group math competitions organized by the Department of Mathematics, Pisa University. One example:

Exercise 1. A bat ate 1050 dragonflies on four consecutive nights. Each night she ate 25 more than on the night before. How many did she eat each night? Solve this algebraically.

It is clear that this problem can be solved easily denoting by x the number of dragonflies that the bat has eaten in the first night. Then we have the simple equation:

$$
x + (x + 25) + (x + 50) + (x + 75) = 1050
$$

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Figure 4: Problem for Bats and Dragonflies reflection and refraction – not *instead* of the actual experiment (if one sees experiments only in

so $4x + 150 = 1050$ and we find $x = 225$.

During the work of the Math Lab in Pisa a specific game with Flash script was compared with the above examples. The game is connected with the following Diophantine equation butting the work of the math hap in risa a specific game with riash script was compared w

$$
Bx = A + Cy,\tag{3}
$$

where A, B, C are natural numbers that we choose before starting the game, while x, y can be interpreted by two buttons (the green on Figure [5](#page-3-0) and the red one on the same Figure). near the centre of the lens. With thicker lenses and light being more off-centre, the calculations where A, B, C are natural numbers that we choose before starting the game, while x, y can

Figure 5: Game with Diophantine equation

If one presses the green button this corresponds to the operation

$$
x \Rightarrow x + 1,
$$

or

$$
Bx \Rightarrow Bx + B.
$$

If the red button is pressed, then the following action **Modelling optical lenses with Dynamic Geometry Software**

$$
y\Rightarrow y+1,
$$

or

$$
Cy \Rightarrow Cy + C
$$

is executed. For this once the prof introduces the numbers B, C, A , the student (player) chooses his strategy: pressing $x - -$ times the green button and $y - -$ times the red one, the program computes $Bx - Cy$ and if this number coincides with A then the exit is successful and the number of all actions d by removing α for piece and putting another piece in. To see what happens if α $\frac{1}{\sqrt{2}}$ is the current lens and put in the new one. Studients can then observe the new one. Studients can then observe then observe then observe the new operators can then observe the new operators can then observe t

 $x + y$ $\alpha + g$

is presented on the screen.

The score is exactly this number $x + y$ and the purpose of the game with many players is to have minimal sum $x + y$. One example is the choice $B = 3, C = 7, A = 1$ chosen by the prof and $x = 12, y = 5$ with "score" $12 + 5 = 17$ is on the Figure [6.](#page-4-0) light through a lens with the help of dynamic geometry software (DGS). reflection and reflection \mathcal{L}^{in} instead of the actual experiment (if \mathcal{L}^{in} on \mathcal{L}^{in} is \mathcal{L}^{in}).

Figure 6: Game with Diophantine equation

One can find a better solution $x = 5, y = 2$ with a better score $5 + 2 = 7$.

The following link enables one to play this game

Link to the game [Diophantine Game](eqDiofantea4.swf)

There are several questions that have been discussed in the lab:

- find triples A, B, C such that the Diophantine equation has no solution;
- find a necessary and sufficient conditions so the the Diophantine equation has at least one solution;
- find an algorithm to win or to find optimal $x + y$.

The results of this attempt to combine the Italian and Japanese teaching experience are very promising and show the possibilities for further developments. $\frac{1}{1}$

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3 Tasks for further development Modelling optical lenses with Dynamic Geometry Software

Possible argument for further treatment is presented on Figure [7.](#page-5-0)

It is 4 o'clock now. When the 2 arms meet each other first?

Figure 7: Clock problem

Possible discussions and solution can be found by using the following.

Hint. Each minute each of the arms makes a rotation in clockwise direction of **HINT.** Each minute each of the arms makes a rotation in clockwise direction of

 $Longarm: 360^0/60 = 6^0,$ $Shortarm: 30^0/60 = 0.5^0.$ ω ing and it does work well only with the summation ω b_{max} more complex, and from the equations alone it would be different.

Thus, each minute the angle between two arms will diminish by

 $6^0 - 0.5^0 = 5.5^0$. ϵ_0 even for the ϵ_0

The angle between two arms at 4 oclock is 120^0 . Therefore,

 $120/5.5 = 21.81818181⁰$.

Answer. the angle of incidence (between the ray of $n = n$) is equal to the angle of reflection: $\frac{1}{n}$ is equal to the angle of reflection: $\frac{1}{n}$ is equal to the angle of reflection: $\frac{1}{n}$ is equal to the angle of reflec

 $4:21:49.$

Another argument introduced and discussed in the problem posing lab is presented on Figure [8.](#page-5-1)

Figure 8: Door problem

Formally, participants of the lab had the following suggestion: try to look around you and introduce some math model and problem. One of the most frequently used vehicles in Pisa are

Dyna MAT

buses of an old generation having a system to open the door presented on Figure [8.](#page-5-1) In some sense the problem can be connected with one of the several examples that have been presented to the participants of the lab. Among these examples we had the following problem taken from Chapter "Astronomy and instruments to draw quadratic curves" in [\[2\]](#page-6-0). **Modelling optical lenses with Dynamic Geometry Software**

Exercise 2. (see chapter "Astronomy and instruments to draw quadratic curves" in [\[2\]](#page-6-0)). A **EXECTENCE:** (SEC ENGPLET TESTORIORS) and most amends to araw quadratic carecter in $[2]$). It student of the famous Galileo Galilei discovered a new planet, which orbits the Sun on an elliptic orbit with semi-axes a and b. Suppose that an observer is located at a point in the plane of the ellipse, such that the ellipse is seen from that point at an angle of $90^{\overline{0}}$. Compute the distance between the center of the ellipse and the observation point. ω is different. It is difficult to some and suppose the smooth in any other way ω is dust on an empty

One can use Cartesian coordinates and find an analytic solution describing by an equation the set of points such that the ellipse is seen from that point at a right angle. Problems of this type can be seen in different places. One can see for example in $[4]$ for some other attractive examples connected with conics. Since the practical experience of pupils in High School is not at very high level, we insisted in the problem posing lab to continue the line from [\[1\]](#page-6-1), [\[2\]](#page-6-0). One can use GeoGebra for the problems [\(2\)](#page-6-2). However, Japanese colleagues that have no information about the existence of GeoGebra tried another approach based on everyday IT tools used in their class anom teaching. The Power Point application is a real animation prepared with Microsoft Office 2010 the can be seen from any free power point viewers. light the glass surface of an optical length and part of its reflection of its reflection of its reflection angle of its reflection and certain angles reflection and certain angles reflection and certain angles of the cert

Here is the link to the ppt file [Power Point application](bus1.ppsx) Disadvantage is the fact that one needs Power Point (even older versions) that is not open source software. the centre of the centre of the lens of the lens of the calculation being more of α calculations of α calculati

The next step was to treat the Door Problem described on Figure [8.](#page-5-1) It is interesting that the first approach of the Japanese colleagues was based on the Power Point animation.

However, the first of the tasks to use GeoGebra applications was done exactly for the Door problems etc. But even for the DGS, we need mathematics to create the simulation in the first place. **2** Expressed by Ω **EXPRESSED surface**

[The link to the Geogebra file is here](door.ggb) **2.1 Reflection**

This example enables one to compare the capabilities of Geogebra and Power Point and see the This champion chasses one to compare the capacimetes of exception and I since I since and see the angle of incidence (between the ray of light and the *normal*) is equal to the angle of reflection:

Other possible tasks are presented by the student Eva Cricca in the Power Point presentation and the Geogebra applications

- optimize the distance between two points, when the obstacle is a river, one can see the [The link to the Power Point file is here](Problema del fiume.pps)
- optimize the distance between two points when the obstacle i a lake (ellipse). and the [The link to the Geogebra file is here](ellisseeva.ggb)

References

- [1] Georgiev, V., Mushkarov, O., Ulovec, A., Dimitrova, N., Mogensen, A., Sendova, E. MEET-ING in Mathematics, Demetra Publishing House, Sofia, 2008
- [2] A. Ulovec, J.Anderson, S. Ceretkov´a, N. Dimitrova, V. Georgiev, O.Mushkarov E Sendova, ˘ MATH to EARTH 2010. **Fig.1** Reflection of light at a plane surface.
- [3] Hiroku Endou , Algorithmic girl, 2006, Japan. $\frac{1}{\sqrt{2}}$ between funded with support from the European Commission in its Lifelong Learning Programme $\frac{1}{\sqrt{2}}$

Dyna MAT

- [4] J. Šunderlk and E. Barcková, Best spot - investigation with circles, chapter in this book. **Modelling optical lenses with Dynamic Geometry Software** Andreas Ulovec
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