Geometry on Car Wheels

Miroslava Sovičová, Štefan Havrlent, Ľubomír Rybanský

1 Introduction

Introducing a new mathematical idea to pupils is often a big challenge for a teacher. The choice of the situation representing the mathematical idea should capture pupils' interest, increase their inner motivation and encourage them to get voluntarily and enthusiastically involved into the classroom work. One of the possible techniques to achieve this effect is to approach mathematical ideas by stimulating pupils' curiosity about things within their perception, within their every-day life and environment. The aim of this article is to introduce the mathematical idea of regular polygons and the concept of a tangent through the car wheels.

Cars are an inseparable part of our lives, they can be found almost everywhere. Pupils are familiar with them, no matter whether they are boys or girls. Car wheels suitably and naturally represent the characteristics of regular polygons and also the concept of tangents to the circle. Therefore, the use of this context can be useful for creating and building of intuitive knowledge about regular polygons and tangents for children in the second grade of primary school.

2 Car wheels as regular polygons

The most commonly used definition of a regular polygon is that it is a polygon which is equiangular (all angles are equal in measure) and equilateral (all sides have the same length). Within our context of car wheels and the spokes on the car wheels, we work with the sides of the polygon and with the distance between the centre of the polygon and one of the vertices. Therefore, for our situation, a more suitable characteristic of a regular polygon equivalent with the previous one is: A polygon is regular if and only if its sides are equilateral and all its vertices have the same distance to the centre of the polygon.

Remarkably, it is possible to find a different number of spokes on the car wheels. During the longer observation of the cars in the city, we managed to find the cars with three to twelve spokes on the wheel. There were also several car wheels with a higher number of spokes.

2.1 Euclidean constructions of some regular polygons

Some of the regular polygons can be constructed only with the straight-edge and compass. To present these tasks for pupils in an innovative and unconventional way, the constructions are suggested as the tasks to re-draw the car wheel from the picture or, possibly, from the car park in front of the school into the worksheet.

Task 1

In the picture (Fig. 1), you can see a car wheel with six spokes. Re-draw the wheel with the spokes into your worksheet. The diameter of the wheel rim is 12 centimetres.

Fig.1 The car wheel with six spokes.

Solution:

For pupils, it is important to realize that the spokes of the car wheel are of the same length. They are the line segments with one end point in the centre of the wheel disc and another one on the rim of the car wheel. The end points of the spokes lying on the rim are, apparently, concyclic points with the same distance between each two adjacent points (Fig. 2).

Fig.2 The regular hexagon on the six-spoke car wheel.

It means that the task for the pupils is to construct the regular hexagon inscribed inside the circle with the diameter of 12 centimetres. It is Euclidean construction only with the use of the compass. The line segments connecting the centre of the regular hexagon with each of its vertices, together with the circle circumscribed about the regular hexagon create the car wheel the pupils are supposed to re-draw, in Fig. 3 marked in blue colour.

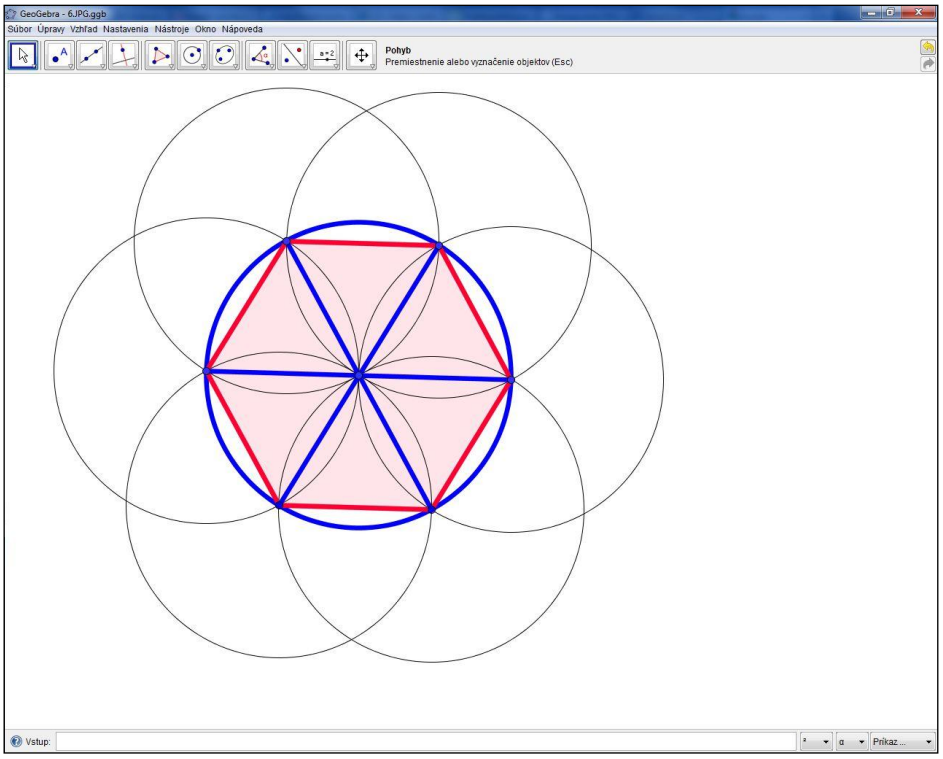

Fig.3 The regular hexagon constructed in GeoGebra.

Task 2

In the picture (Fig. 4), you can see a car wheel with three spokes. Re-draw the wheel with the spokes into your worksheet. The diameter of the wheel rim is 10 centimetres.

Fig.4 The car wheel with three spokes.

Solution:

The geometrical situation "hidden" behind the three-spoke wheel task is to inscribe the equilateral triangle inside the circle with the diameter of 10 centimetres and to find its medians. The centre of the circle is the centroid of the triangle – the point where the medians of the triangle intersect.

Figuratively, the spokes of the car wheel are the longer parts of medians divided by the centroid (Fig. 5).

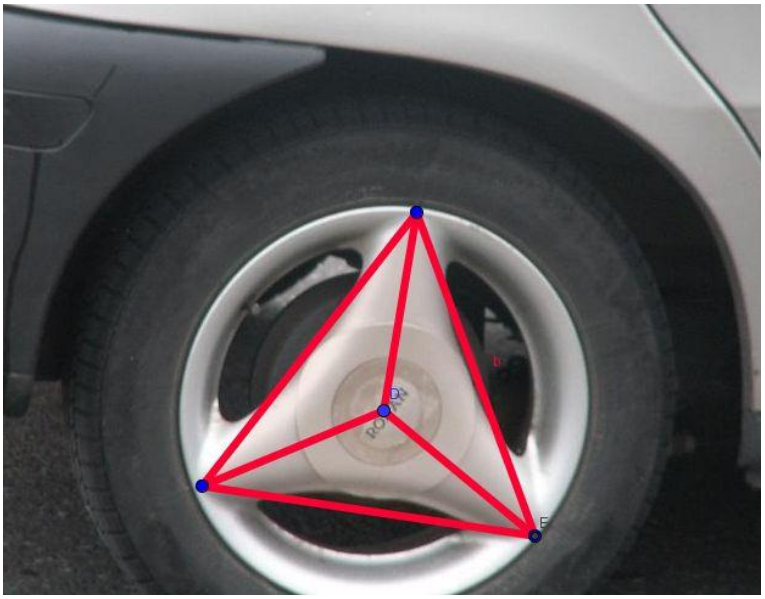

Fig.5 The equilateral triangle on the three-spoke car wheel.

In Euclidean construction of the equilateral triangle inscribed inside the circle with the diameter of 10 centimetres, we use the circle with the same diameter, the perpendicular line and the properties of medians and the centroid of the triangle. Similarly, the line segments connecting the centre of the equilateral triangle with each of its vertices, together with the circle circumscribed about the equilateral triangle, create the car wheel the pupils are supposed to re-draw according to the task (in Fig. 6 marked in blue colour).

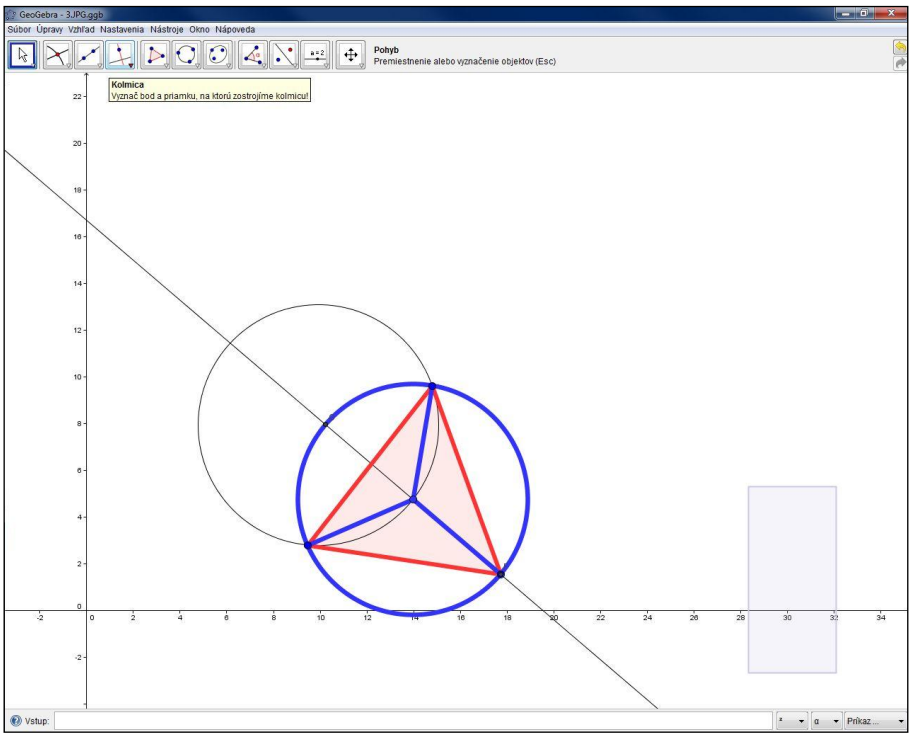

Fig.6 The equilateral triangle constructed in GeoGebra.

In a similar way, the tasks with the same assignment for other regular polygons which are constructible by Euclidean construction can be created. They vary in the level of difficulty, and, therefore, can be used not only in primary or secondary schools, but also as the task for university students of mathematics.

Fig.7 Other regular polygons on the car wheels.

2.2 The axis symmetry of regular polygons

Depending on the number of vertices, the regular polygons have several axes of symmetry. If the number of vertices is odd, all the axes pass through a vertex and a midpoint of the opposite side. If the number of vertices is even, one half of the axes passes through two opposite vertices and another half of vertices passes through the midpoints of the opposite sides of the polygon.

The tasks for axis symmetry of regular polygons on the car wheels are suggested in a way that supports the gradual and subconscious awareness of the above-mentioned propositions.

Task 4

Complete the pictures of the car wheels as accurately as you can. Keep the right distance between the spokes.

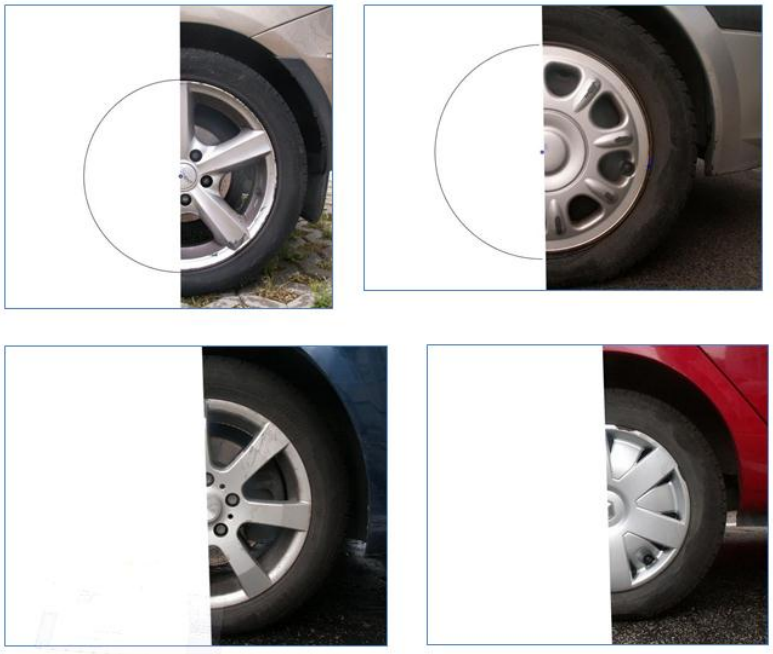

Fig.8 Regular polygons for the axis symmetry.

Solution:

For the most precise and accurate completion of the task, the pupils need to realize that they should work with the spokes visible in the part of the picture as with the halves of the axes of symmetry of regular polygons. The pictures in the task are chosen to cover the odd and also the even number of the vertices, so the pupils should differentiate between them when completing the pictures with the help of axes.

3 Tangents

While driving the car, it sometimes happens that a small stone flies off the wheel of the car in front of you and hits your car (its front mirror, or, possibly, the front part of your car) and can damage it.

Task 5

In the picture, there is your car and a wheel of a car driving in front of you. Draw a fender to the wheel of the car in front of you, so that the stones flying off the back wheels do not hit the front mirror of your car.

Fig.9 The worksheet for pupils to Task 5.

Solution:

The analytical solution of this task is rather difficult for the pupils of primary or secondary schools. However, this problem can be solved easily in a graphical way.

It is a well-known fact from Physics that during the circular motion, even after removing the action of force of the body, the body still moves in a steady linear motion in the direction of the tangent. During the motion of the car, the stone caught in the tire tread pattern of the wheel performs the steady circular motion. In the moment, when the stone flies off, it flies from the car wheel on the trajectory of parabole, influenced by the gravitational field of the Earth. To make the task simpler, we do not consider the gravitational field of the Earth which means that the stone flies off on the tangent trajectory.

The task for the pupils is to construct several tangents to the wheel of the car which moves in front of our car. In such a way, pupils find out which stones can possibly hit the front mirror of the car.

For the exact solution of the task, it is necessary to construct the tangents to the car wheel that goes through the bottom and upper point of the front mirror. The points between the two points of tangency are the area that should be covered by the fender.

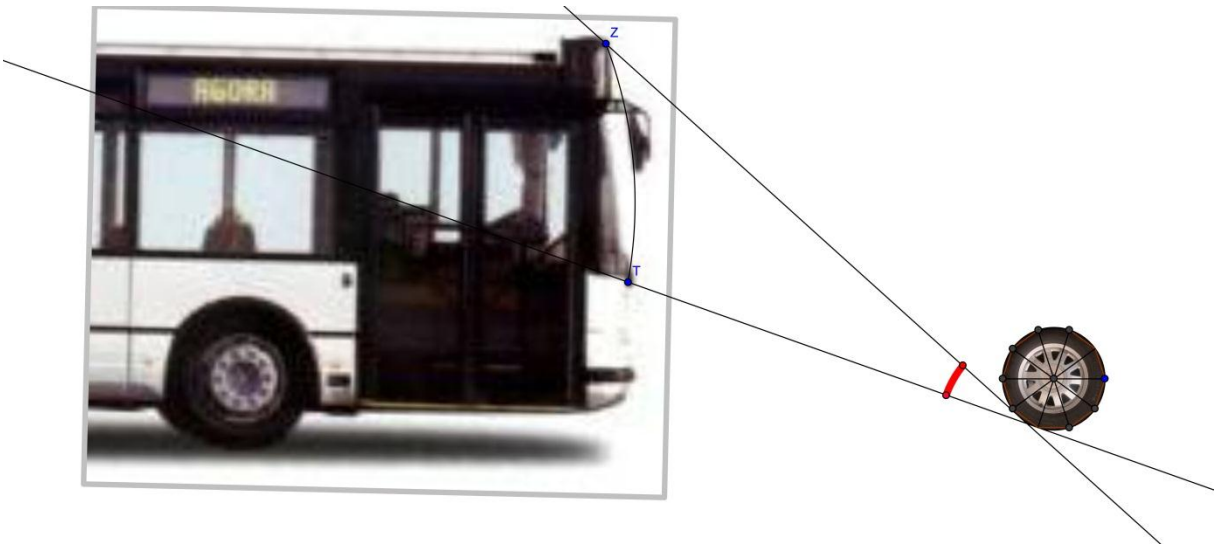

Fig.10 The exact solution of the task.

We created the applet to this task in the geometrical software GeoGebra.

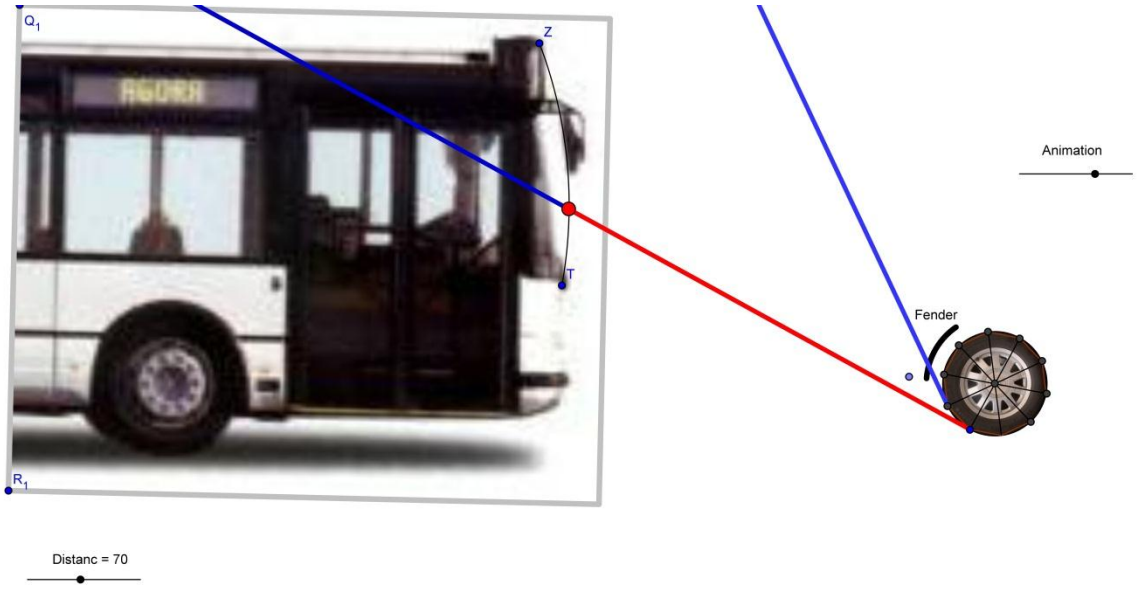

Fig.11 The applet created to Task 5.

4 Conclusion

The tasks in this article were created as an aid and inspiration for the teachers of mathematics for their teaching. The pictures of car wheels can be used in teaching of the various topics from geometry, for example the construction of equilateral triangle, the properties of regular polygons, Euclidean constructions of regular polygons, the axis symmetry and also the significance of the tangents in primary and secondary schools. Moreover, the tasks can be used on methodological seminars for the future teachers of mathematics as the examples of creating own innovative and non-traditional tasks for pupils. The possible extension of this topic can be to use the car wheels for the construction of regular star polygons, such as pentagram or hexagram.

References

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