Creating dynamic geometry constructions as composition tools in art and photography

Evgenia Sendova, Toni Chehlarova
Institute of Mathematics and Informatics, Bulgarian Academy of Sciences

I. Introduction

Seeing is not as simple as it looks
Ad Reinhardt

Many artists claim that they could explain nothing about their works, that their paintings came upon them by inspiration. The founder of the abstract art however, Vassily Kandinsky, expresses in the book "Concerning the spiritual in art" his theory of painting and sums up ideas that influenced his contemporaries. He makes the brave prediction that *we are fast approaching the time of reasoned and conscious composition, when the painter will be proud to declare his work constructive*.

To motivate better the study of geometry for students with interests in art we could reveal for them the strong relation between the esthetics of artistic compositions and some geometric principles. When reading the works of art critics we come across notions such as *harmony, style, rhythm, balance* (not necessarily the better defined *rules, symmetry, geometry*). Perhaps they think that if revealed the "rules" behind a balance composition would trivialize the art. To us, revealing certain patterns and rules would in contrast raise the level of appreciation of an observer. The modern fine art tries to speak about things which *will be seen*, that is why its language is not understandable for many. But this language could be better learned if we try to study it together with the language of geometry.

Let us consider several relatively simple geometric constructions which have proved useful in creating and studying the balance of the fine-art compositions. After describing them we shall implement them in a dynamic software environment (GeoGebra in our case) so as to illustrate how they could be applied to exploring various paintings (classical and more modern alike).

II. Rabatment

A compositional method broadly used in the 19th century was the *rabatment.* This method consists of taking the shorter side of a rectangle and placing it against the longer side (rotating the shorter side along the corner), creating points along the edge that can be connected directly across the canvas as well as a diagonal from these points to the corners. In a rectangle whose longer side is horizontal, there is one implied square for the left side and one for the right; for a rectangle with a vertical longer side, there are upper and lower squares. In traditions in which people read left to right, the attention is mainly focused inside the left-hand rabatment, or on the line it forms at the right-hand side of the image (Fig. 1)

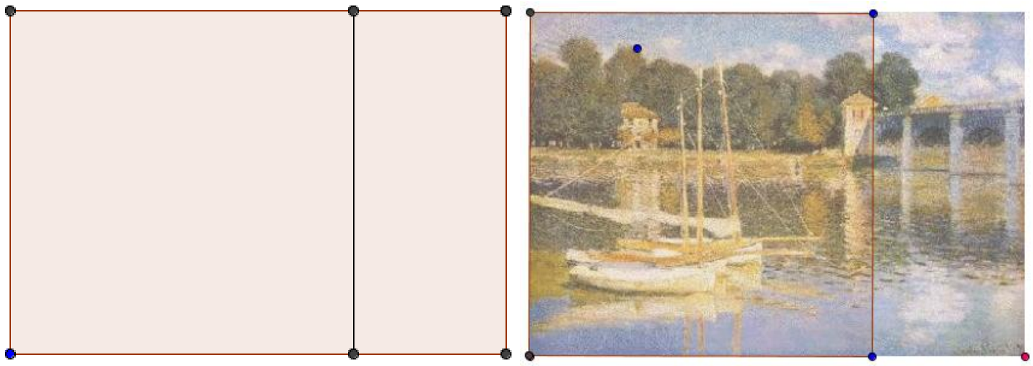


Fig. 1 The left rabatment and its appearance in a Monet's painting

To achieve a more powerful composition one could add the diagonals of the rectangle and the two squares. The focus of attention is clearly seen in the painting of Giotto in Fig. 2

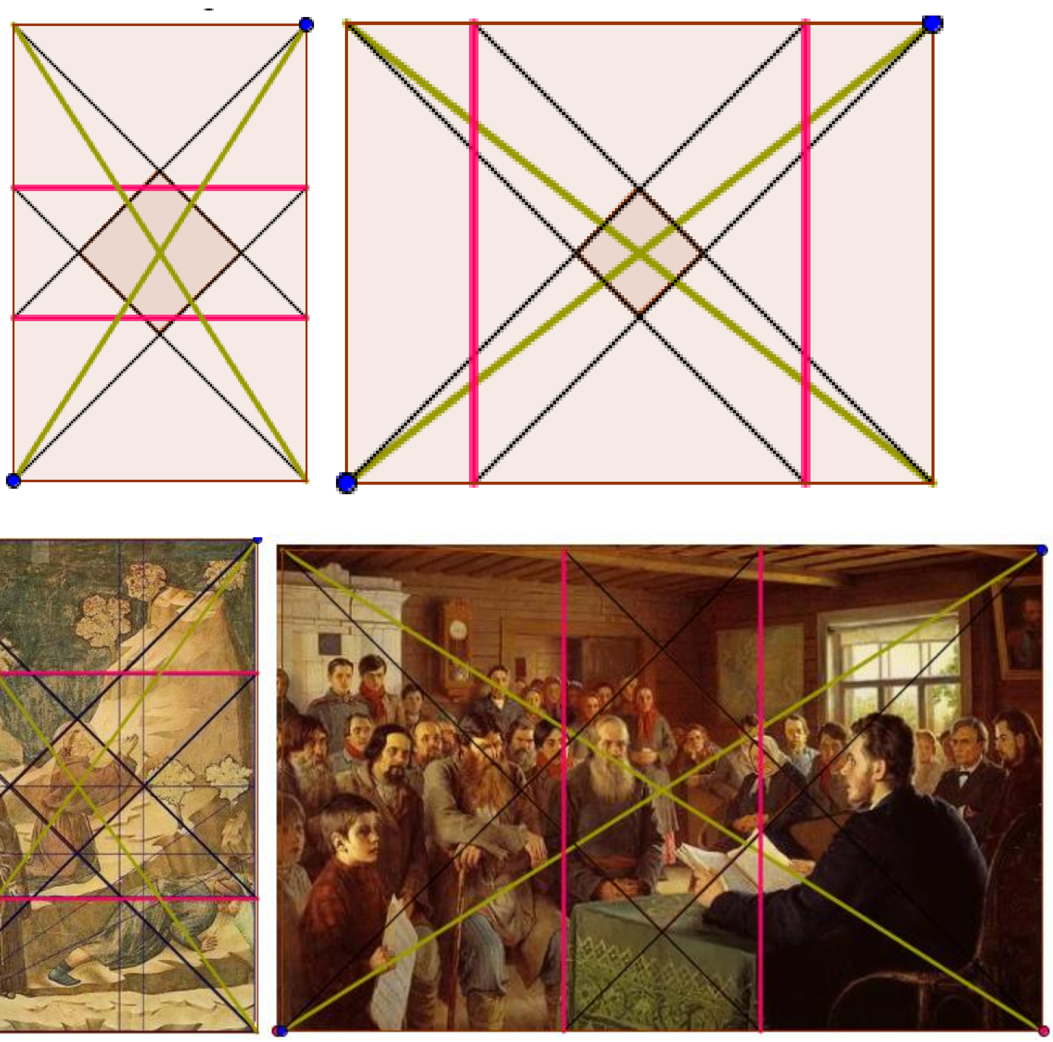
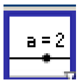


Fig. 2 The rabatment as applied in paintings by Giotto and Bogdanov-Belsky

Let us now create a dynamic rabatment in GeoGebra as a specialised button of its toolbar,

III. Creating a Rabatment button in GeoGebra

We shall start with constructing a dynamic rectangle. This could be done in multiple ways but we shall use two of them which turn out to be the most appropriate for creating a Rabatment button.

First approach – we construct as independent objects the corner of the rectangle (point *A*) and two sliders for its width *a* and length *b* by means of the button  and specifying the interval to be [1, 10].

We define point *B* (the opposite corner of the rectangle) by entering in the command row:

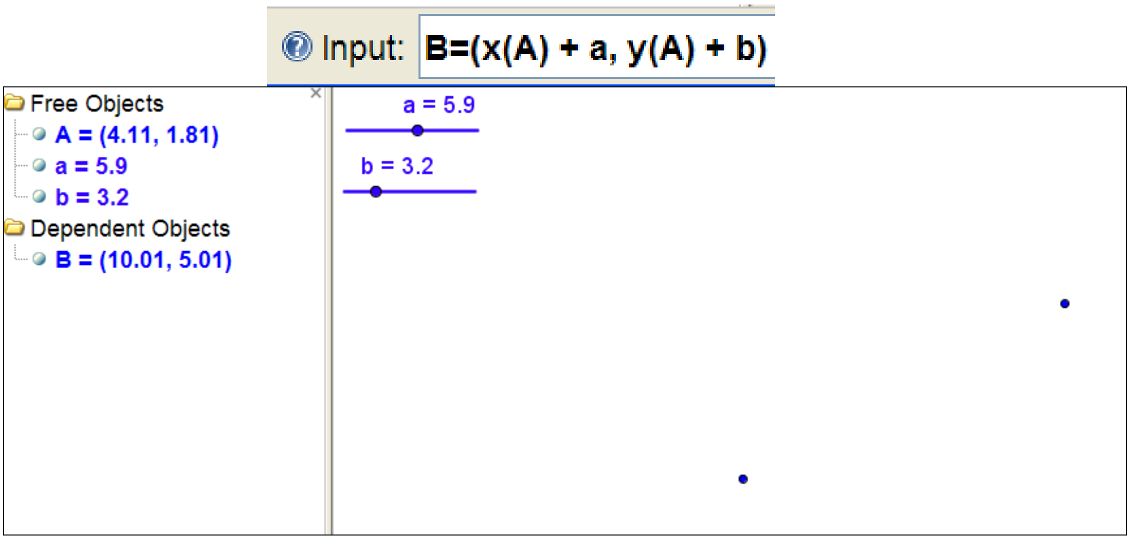
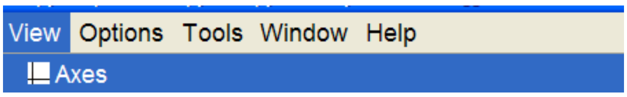
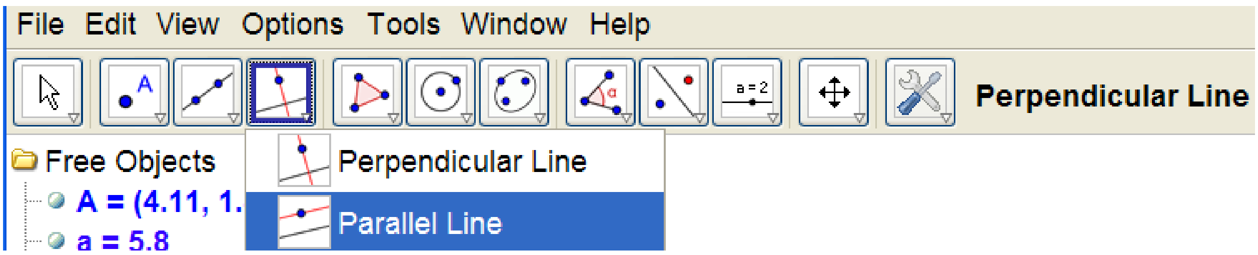


Fig. 3 Constructing two opposite corners of a rectangle

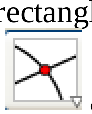
Next we construct lines through *A* and *B* parallel to the coordinate axes by first showing the coordinate system:



and then using the button for a parallel line:

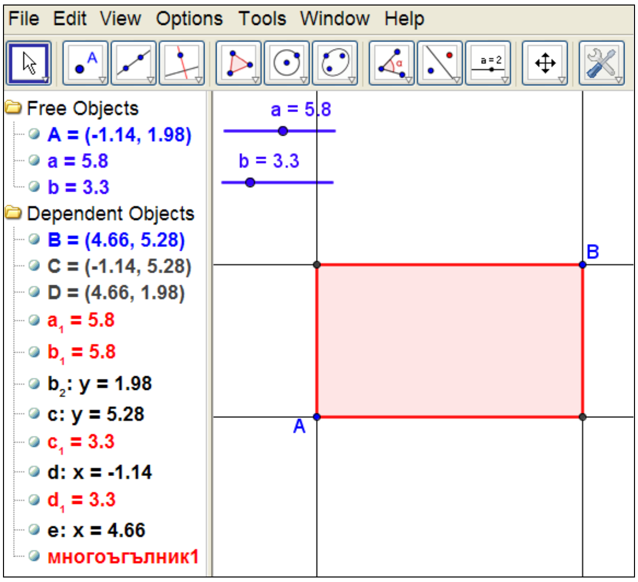


Let us remind that to do this we first click on the point and then on the axis parallel to the line under construction.

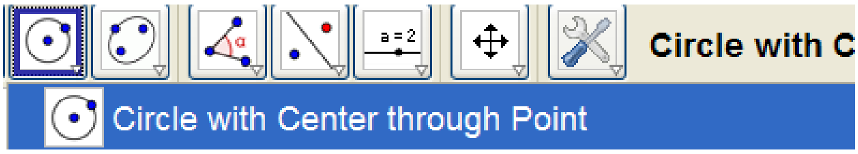
The remaining two vertices of the rectangle could be obtained as intersection points of these lines, i.e. by clicking on the button  and then consecutively on the lines.

Then we construct a quadrilateral by means of the button 

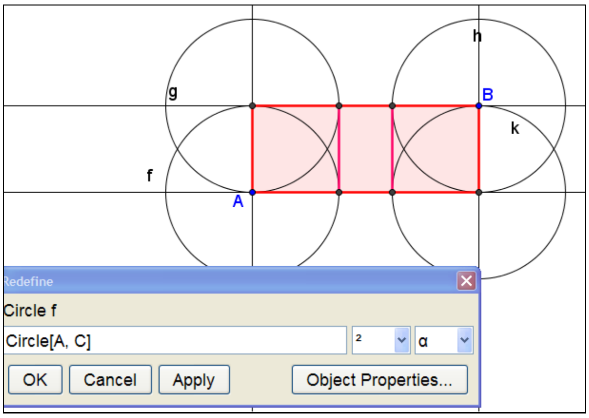
with vertices the four points. Thus we constructed an object (polygon1) with sides the segments a_1, b_1, c_1, d_1.

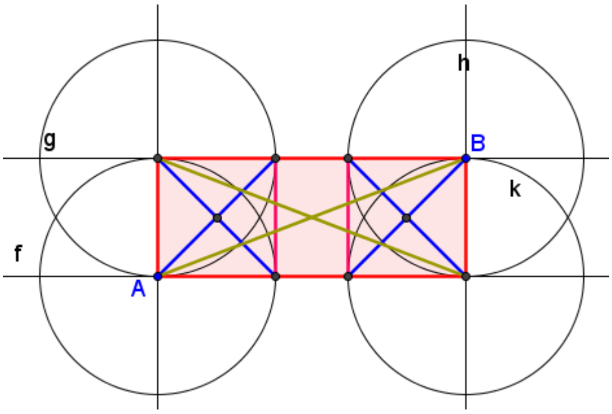


Let $b < a$. Now we construct circles with centers the four vertices of the rectangle and a radius b – the length of the shorter side of the rectangle. We use for the purpose the button for a circle by a center through a point (the point being the other end of the segment *b*):

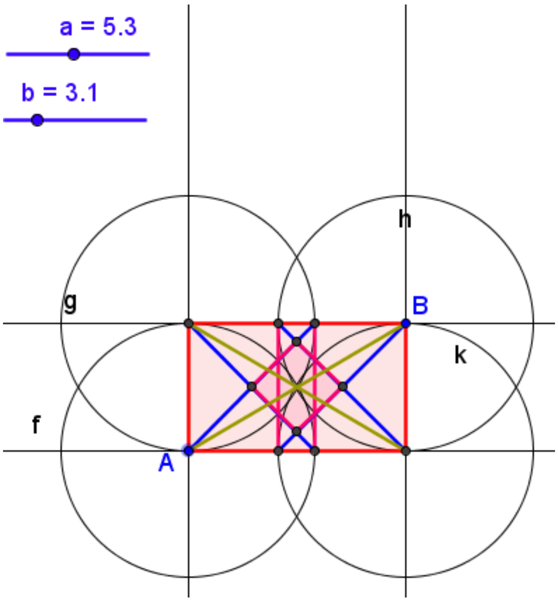


We find the intersection points of the four circles with a side of the rectangle and construct two of the rabatment segments. Then we complete the construction with the diagonals.

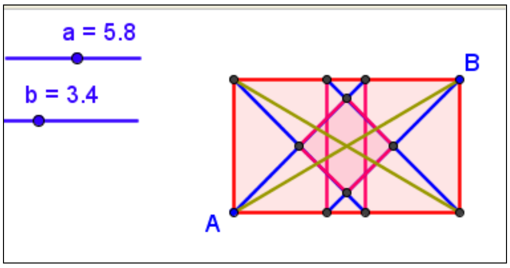


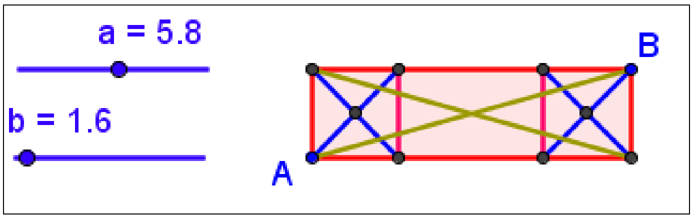


What is left is to construct the square in the center which appears when $b < a < 2b$. (If necessary, we move the slider and construct the intersection points needed and the square.)

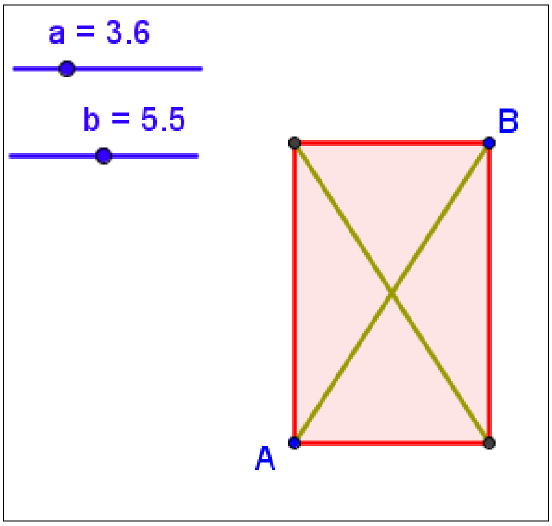


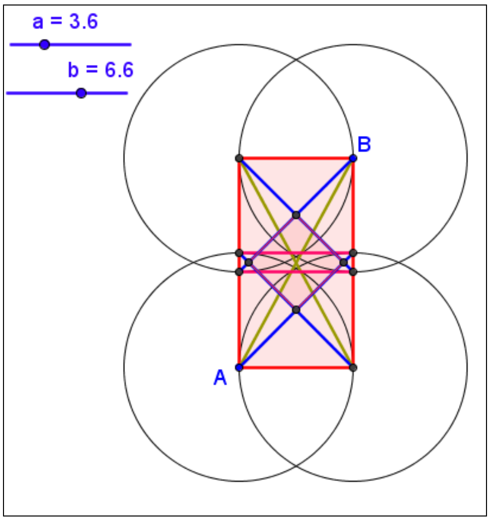
We hide the auxiliary objects (the lines and the circles) and we explore the construction for various values of a and b.





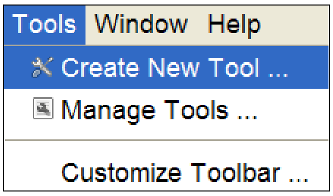
It is clear that if $a < b$ only part of this construction remains. We make similar constructions so as to make a workable construction for a rectangle with a shorter base.



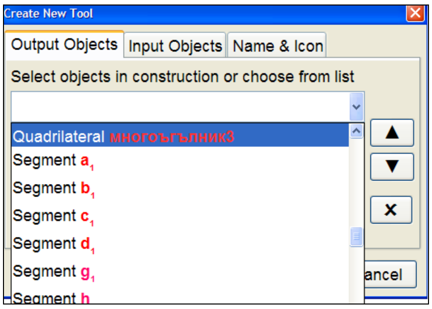


After hiding the auxiliary objects we get a construction ready for explorations.

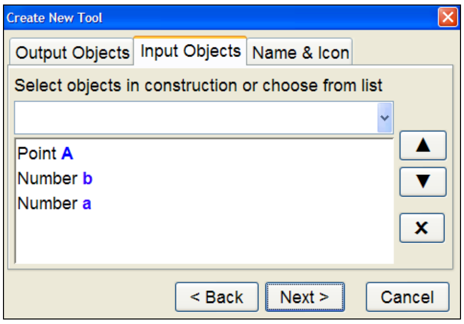
To make the things easier for implementation we shall enrich the tool bar with a Rabatment button. Here it is how we can create a new tool:



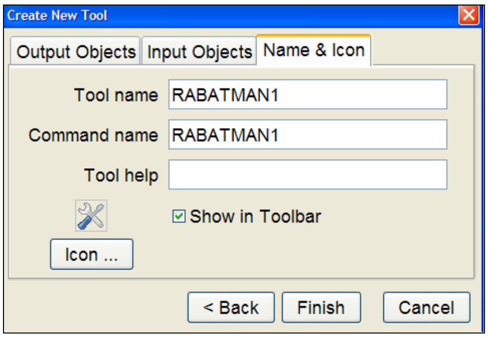
We select the objects of the rabatment construction (segments and polygons):



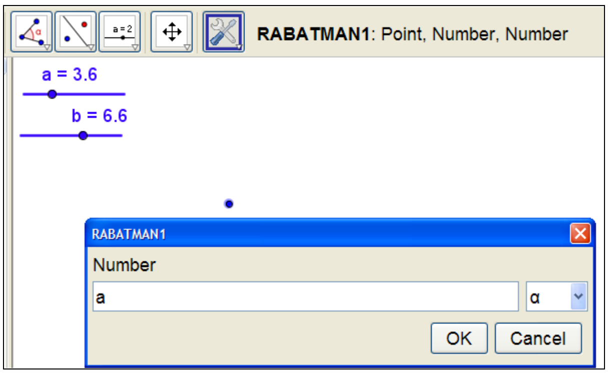
The inputs are inserted automatically (a point and two numbers in our case):



We name the button (later we could make a suitable icon):

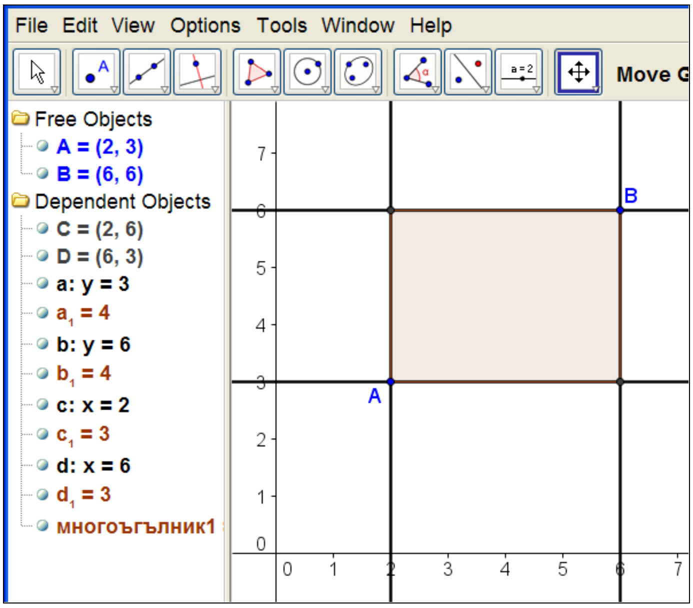


The button has been successfully created. After clicking on it we have to enter a point, a number (we shall use the slider a), and another number (the slider b).



Then the rabatment construction appears on the screen.

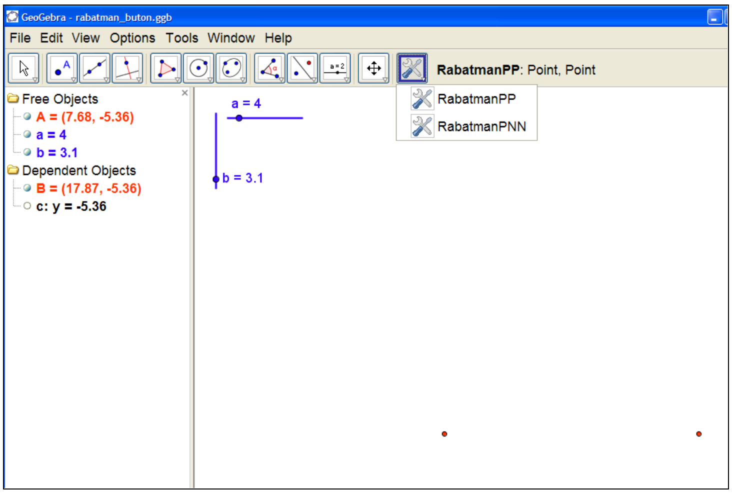
Second approach – two points, the ends of the diagonal of the rectangle, are constructed as independent objects. (If we would like to explore the ratio of some segments, we should additionally display their lengths.) Then we continue similarly to the *First approach* and we construct a rectangle in which the points *A* and *B* are free (movable) points.



It is convenient to construct two Rabatment buttons in the same GG file (named RabatmanPNN and RabatmanPP after the necessary inputs for the respective construction – a point and two numbers in the first case, and two points – in the second).

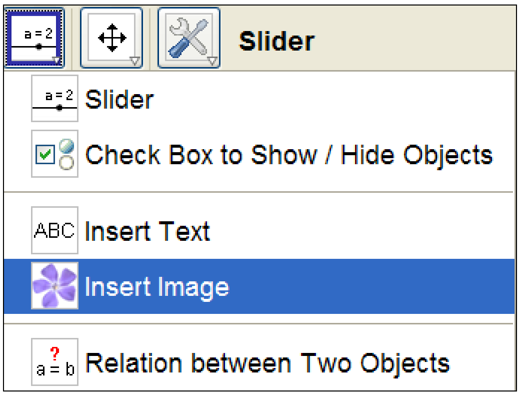
Then we delete the constructions.

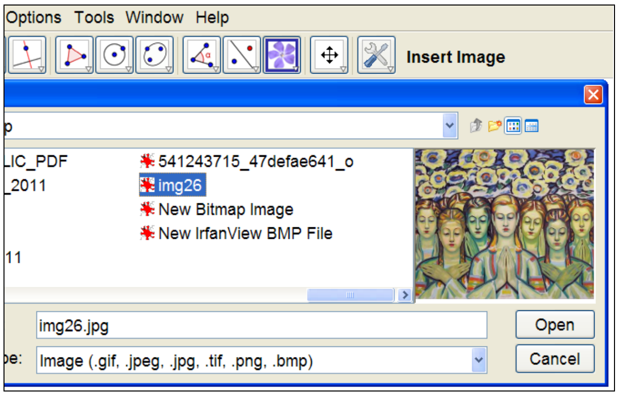
In addition, we construct two sliders for the parameters to be used with the Rabatman PNN button, and two points to position the images.

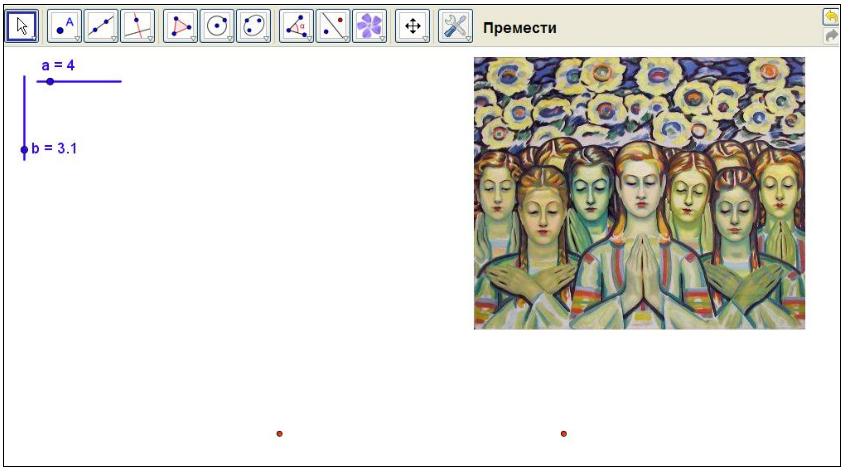


IV.Inserting images on the GeoGebra screen

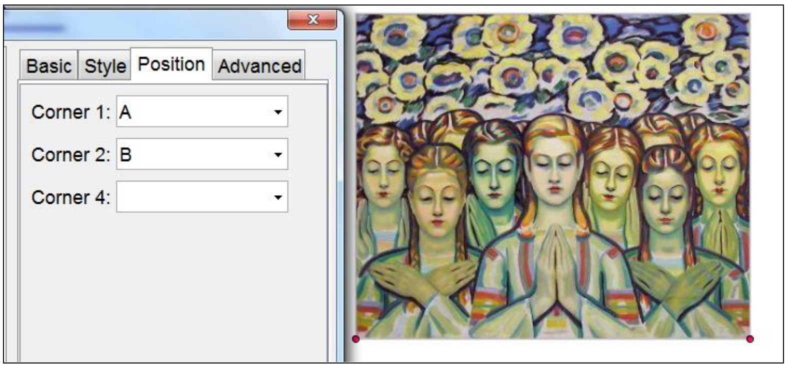
In order to explore a specific painting (saved preliminary in a graphic file) we shall first display it on the GeoGebra screen by means of the *Insert Image* button:



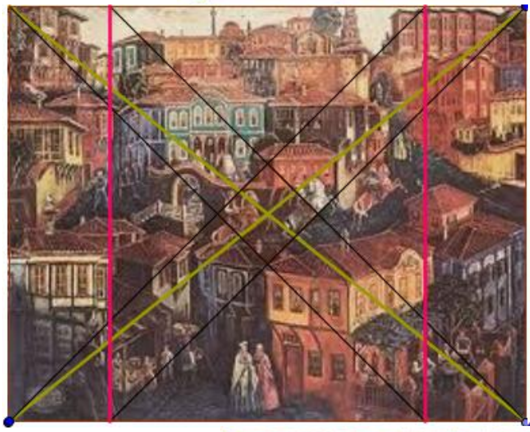


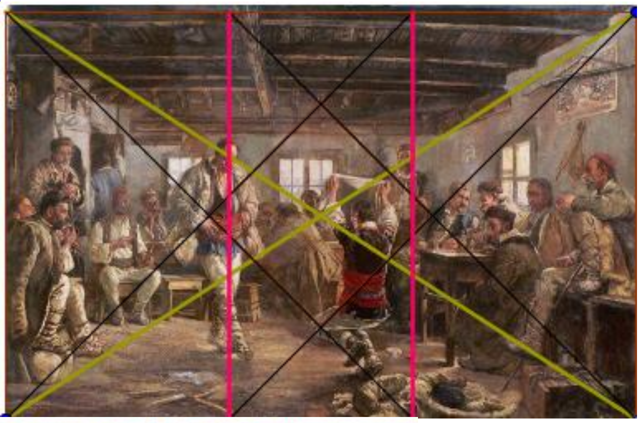


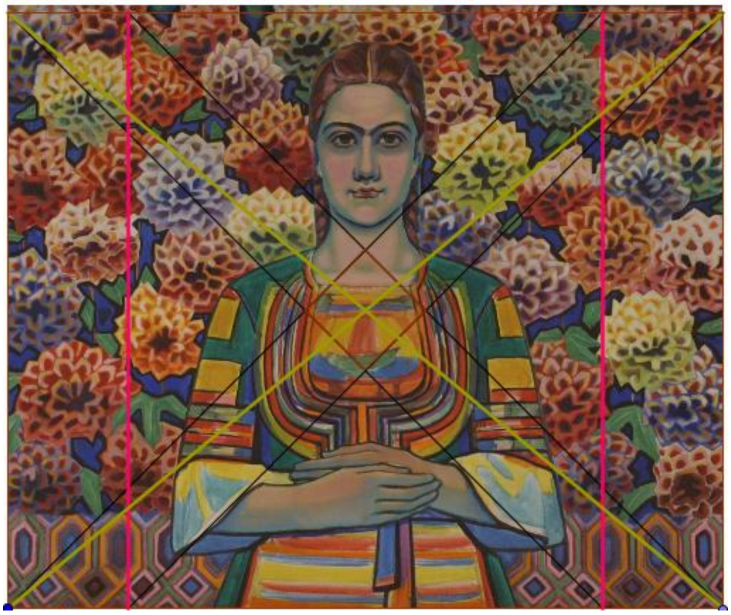
To be able to resize the inserted image by preserving its proportions it is convenient to associate two of its vertices with two preliminary constructed points, e.g. A (a free object) and B – lying on a horizontal line through A (parallel to the X axis).



Then we can use the Rabatment button, place the rabatment construction on the image and look for interesting properties of the composition. Here are some experiments with works by one of the most talented 20th century Bulgarian artists Vladimir Dimitrov – the Master.





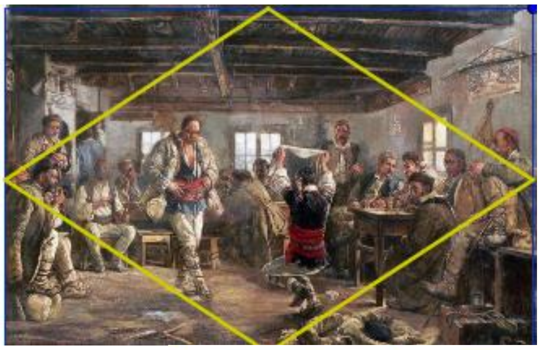


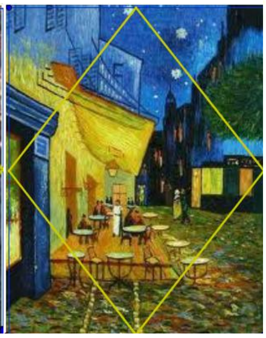
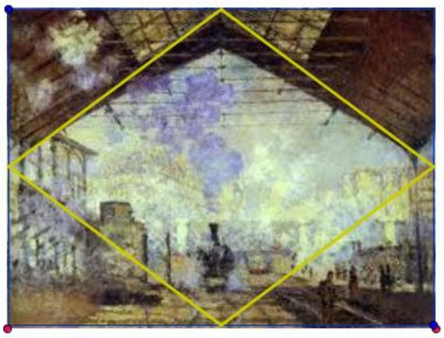
Now you might want to explore the rabatment composition method in the works of such famous artists as Delacroix, Ingres, David, Degas, Sargent, Henri, Cassatt.

V. Some simpler geometric constructions for exploring art composition

a. The central rhombus

The main idea (the logical emphasis) of a painting is often located in a rhombus with vertices the midpoints of the sides of the rectangle:

Task 1. Construct a rhombus button in GeoGebra for exploring images.
Hint. Start with the construction of a rectangle similarly to the Rabatment construction.

b. The rule of thirds

The *rule of thirds* is a simple method that can be used not only as a tool for exploring the paintings of famous artists but also to enhance and improve our own compositions (when we draw or take pictures).

In the diagram below, a rectangle has been divided horizontally and vertically by four lines. The rule of thirds states that the points of interest for any rectangle is determined by those lines. The intersections of the lines are considered by some specialists to be *power points* (the black dots in Fig. 3).

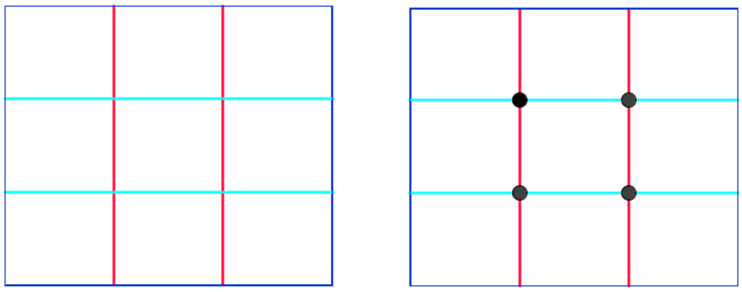
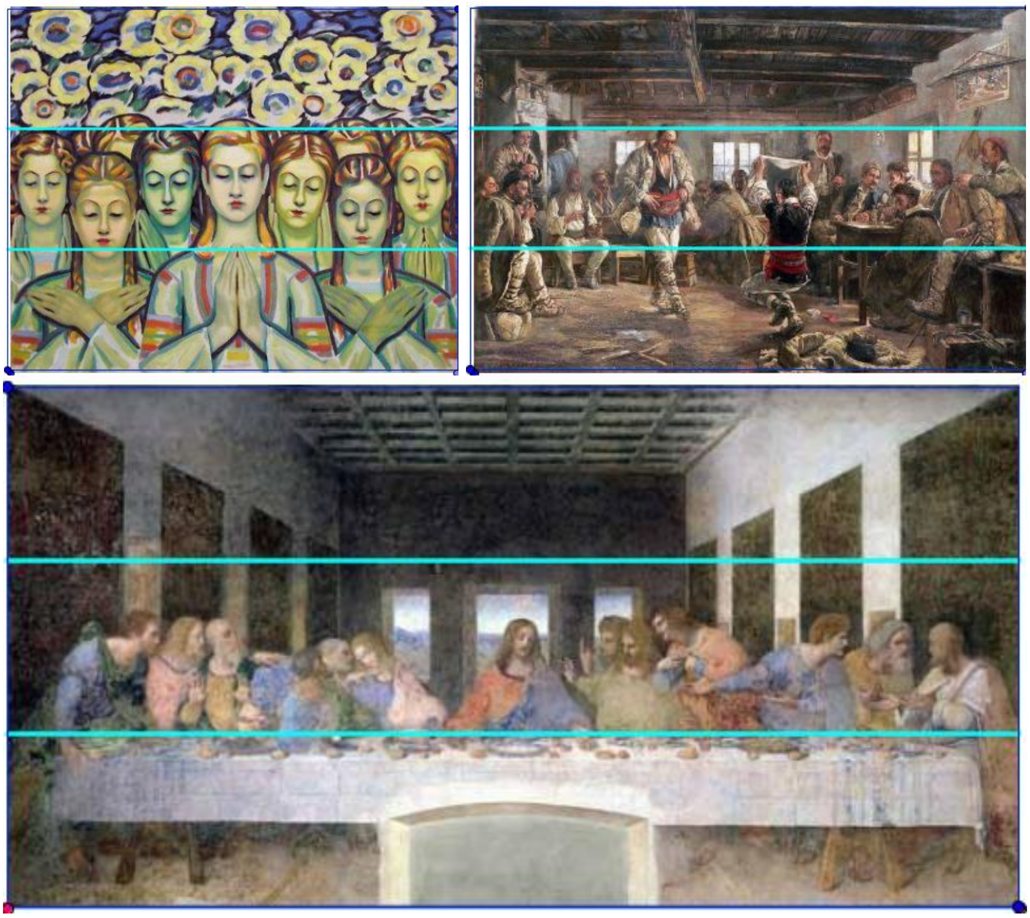


Fig. 3 The rule of thirds and the power points

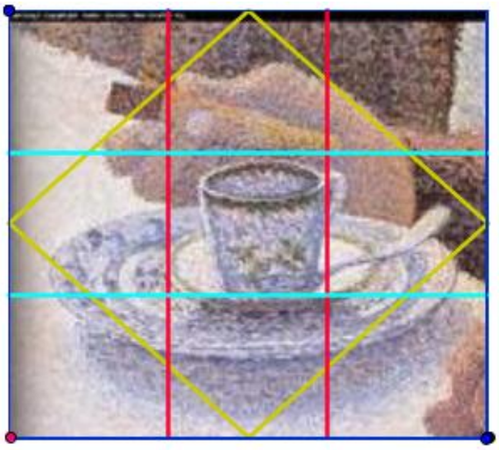
Here is the rule of thirds in action (in horisontal version):

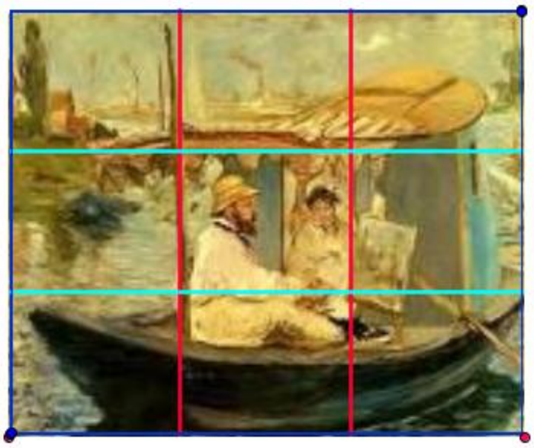


And in vertical one:

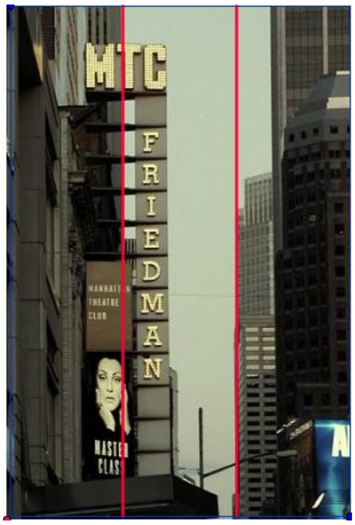
In both paintings above the woman's head is located within the central third of the rectangle but the positions of the hands define different orientation of the frames.

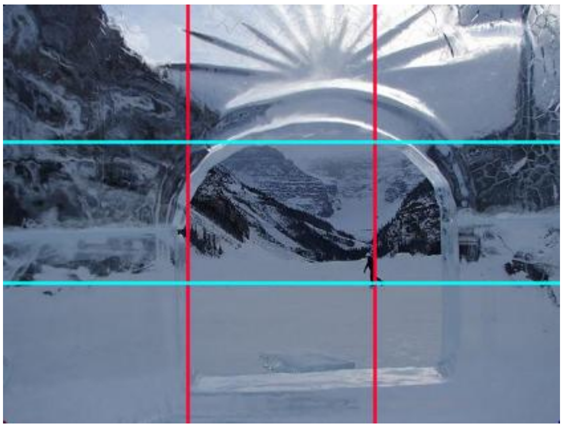
When applying simultaneously the vertical and the horizontal lines we get a central square which often plays a special role. Here are examples of two French impressionists:

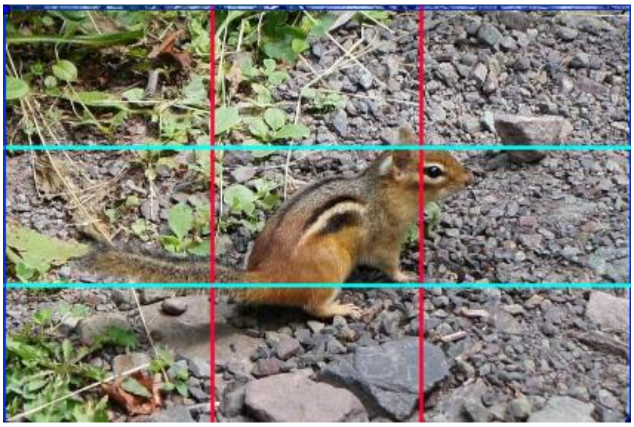




The rule of thirds could be applied easily when making photographs of a scenery - putting the horizon 1/3 of the way from the top or 1/3 of the way from the bottom creates a more attractive composition. Similarly, this rule works with vertical elements instead of horizontal ones.





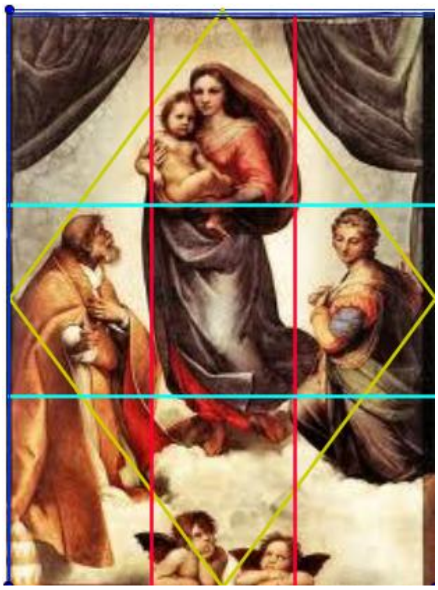
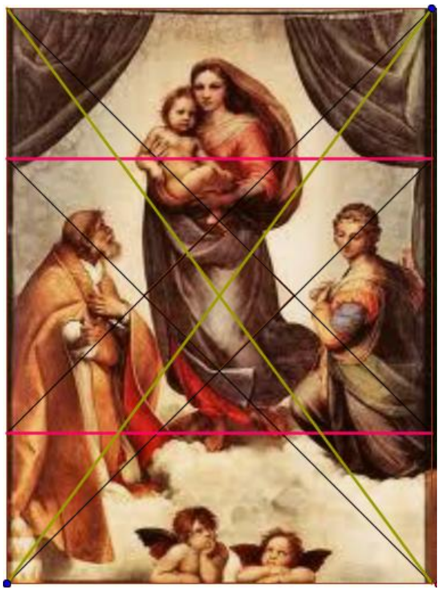


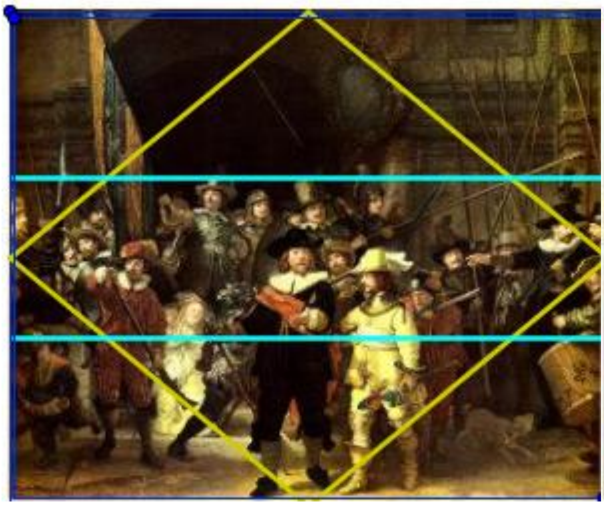
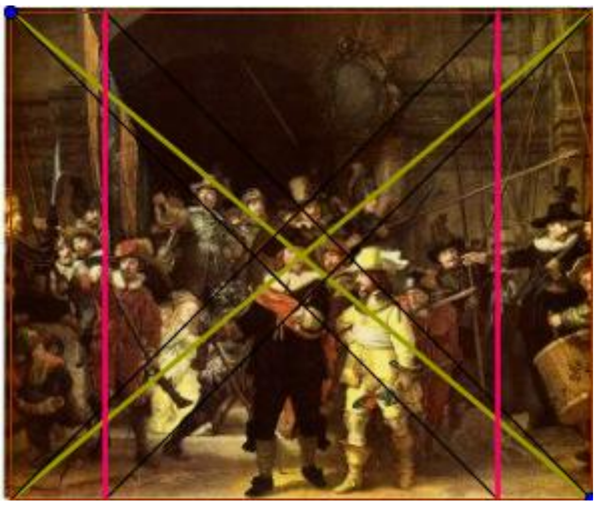
Task 2 Make several digital pictures of a scenery by applying the rule of thirds in just one of them. Explain which version seems the most balanced one.

Task 3 Create *Thirds* buttons (a vertical and a horizontal versions).

Task 4 Explore some classical and some modern paintings by all the composition buttons.

Hint. Insert images from the virtual galleries on the Internet following the instructions in Section IV. Use consecutively the different composition buttons, e.g.

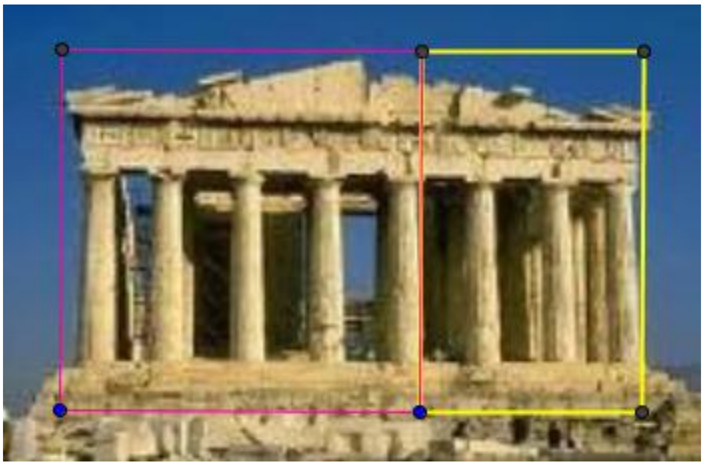



VI. The golden section in art

The most famous mathematical compositional tool, though, is the *Golden Ratio*, also known as the *Golden Mean* or the *Golden Section*. It is defined as the point at which a segment can be divided in two parts *a* and *b*, so that the ratio of the longer part (*a*) to the whole (*a*+*b*) is the same as that of the shorter part (*b*) to the longer one**,** i.e. $a/a+b = b/a$:

This ratio is denoted by Φ. A rectangle whose side lengths are in the golden ratio Φ is called a *golden rectangle*. The golden rectangles could be found in the compositions of artists and architects throughout history. Here are some examples:



We usually find the golden ratio depicted as a single large rectangle formed by a square and another rectangle. What is unique about this is that we can repeat the sequence infinitely and perfectly within each section (Fig. 4).

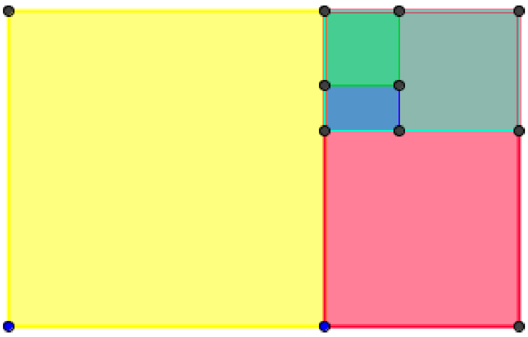


Fig. 4 Golden rectangles

If we take away the big square on the left, what remains is yet another golden rectangle…and so on.

Task 5. Construct a sequence of golden rectangles in GeoGebra (as shown in Fig. 4).

Hint. Fist construct a single golden rectangle as follows (Fig.5):

1. Construct a unit square (blue).
2. Draw a segment from the midpoint of one side to an opposite corner.
3. Use that segment as the radius of an arc that defines the longer dimension of the rectangle.

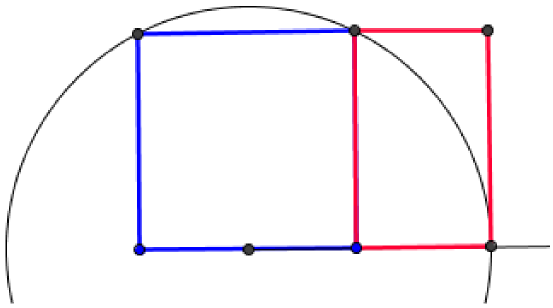


Fig. 5 Construction of a golden rectangle)

Task 6. Construct an arc of 90^{o} in each square so as to get the golden spiral in Fig. 6.

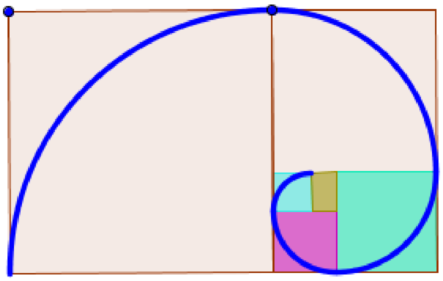


Fig. 6 The golden spiral

Task 7. Construct the *eyes* of the golden rectangle (Fig. 7) and create a corresponding button for locating the *eyes* of a composition.

Hint:

1. Draw the diagonals of the rectangle.
2. From the center to each corner, construct the midpoint of every half-diagonal.

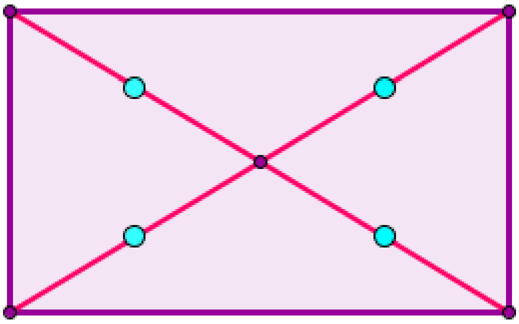
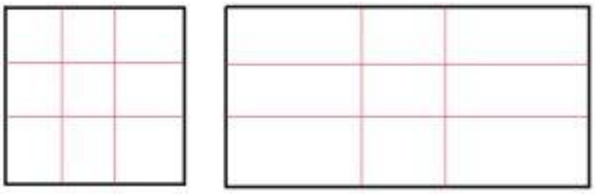


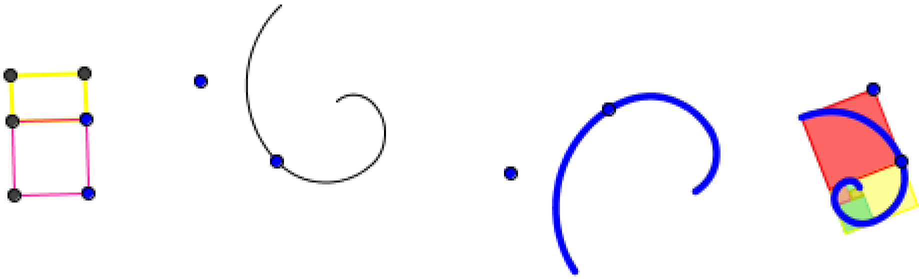
Fig. 7 The *eyes* of a rectangle

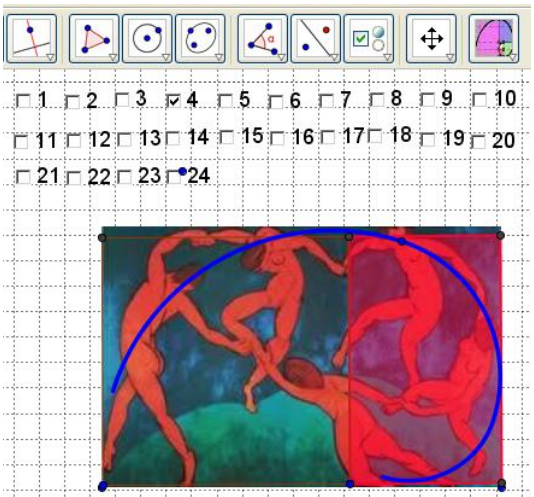
These points—represented by the blue dots in the diagram above—are called *the eyes of the rectangle*.

The placement of the golden ratio intersections varies according to the proportions of the canvas. In the first format below, the golden sections divide the square canvas almost in thirds, whereas in the second one the lines fall closer to the centre:



Task 8. Construct buttons based on the golden ratio and explore the paintings in the folder *Golden*. Describe what you observe.





When describing a painting you might need a more specialized vocabulary. Here is something to start with:

Art vocabulary

Balance – can be symmetrical, asymmetrical, or radial
Emphasis – the part of the painting that draws your attention
Harmony – refers to the interrelationships of elements
Movement – determined by the way the eye moves through the painting
Proportion – the relationships between elements, including ratios such as the Golden Ratio
Rhythm – the placement of elements to create a *visual tempo or beat*

VII. Dynamic mini-projects

1. Take a picture of a scenery in two ways so that they reflect specific goals. Explore the result by means of dynamic constructions and edit the pictures correspondingly by cutting out.

2. Arrange for a picture in two ways (according to two methods for composition):

- 6 persons at a birthday party sitting around a round table
- a class of 24 pupils and their teacher
- flowers and fruits
- perfumes and an advertisement

Explore the result with dynamic constructions and make corrections if necessary.

3. Make an advertisement in two ways of:

- your school
- your hobby
- natural juices
- an old town

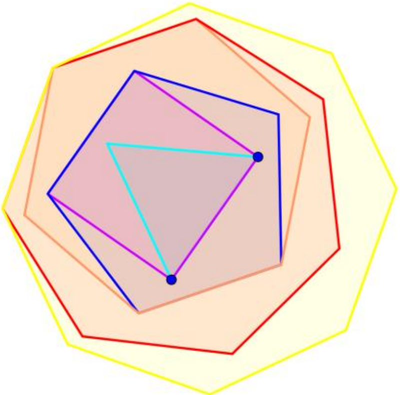
Explore the result with dynamic constructions and make corrections if necessary.

4. Make in two ways a design of an invitation cart for:

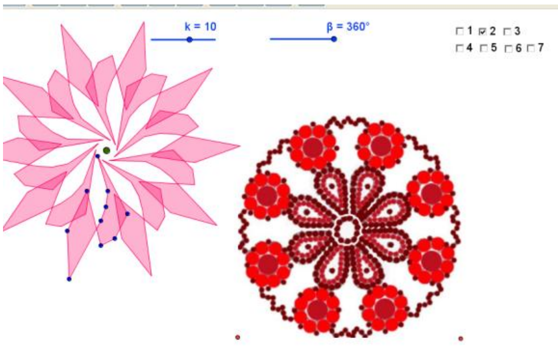
- a fest of mathematics (physics, music, the flowers, athletics)
- a ball with masques
- a birthday party

Explore the result with dynamic constructions and make corrections if necessary.

5. Create a dynamic construction in the style of the artist Max Bill

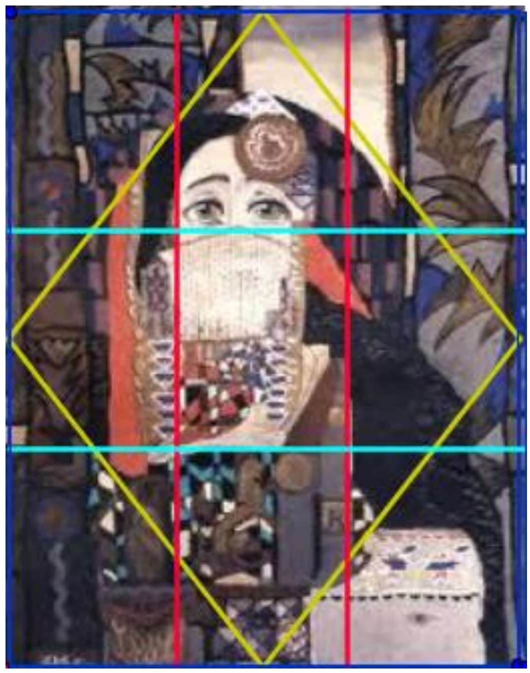


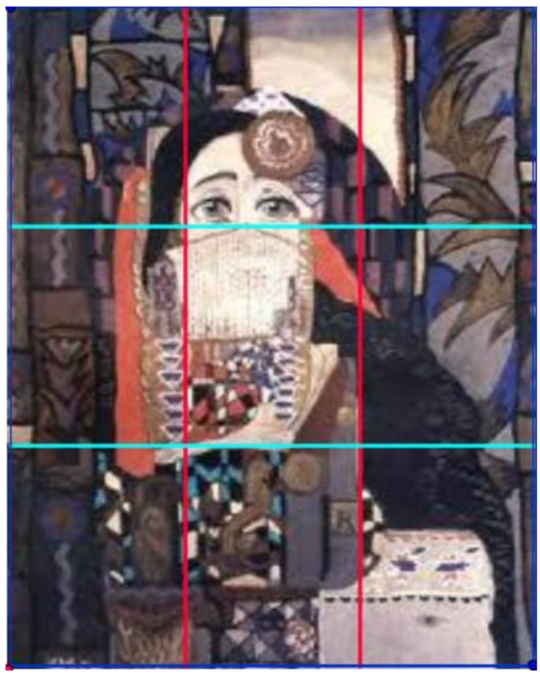
6. Explore the rotational dynamic constructions by means of the sliders so as to create models similar to the pictures of rotational objects.

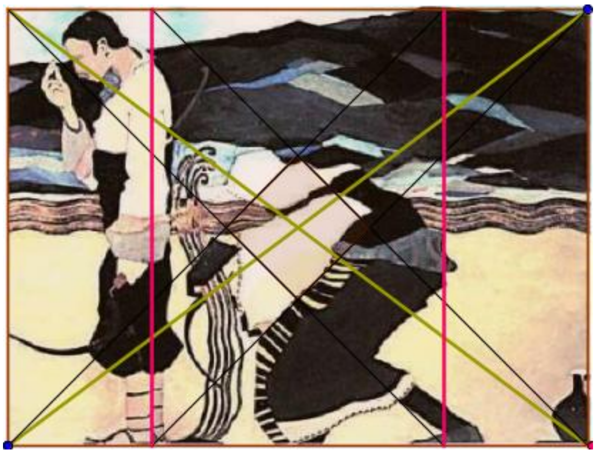


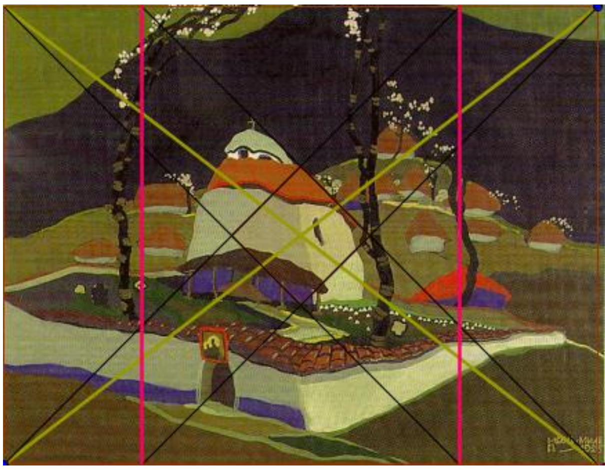
7. Create models of objects around you based on rotational symmetry (wood carved ceilings, embroidered table clothes, etc.)

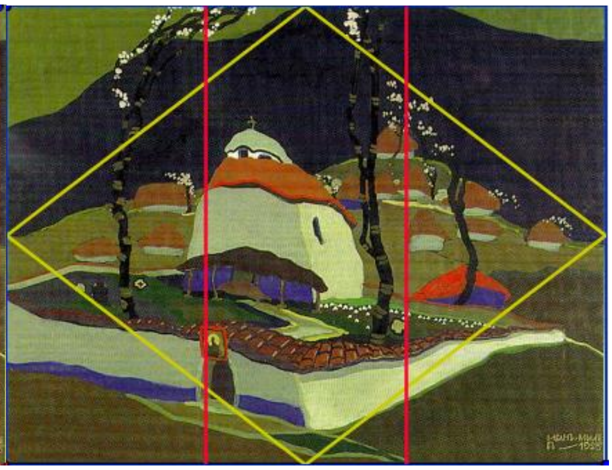
8. Find paintings by the Bulgarian artist Ivan Milev and explore them by means of dynamic tools.
Hint: http://www.ivan-milev.com/

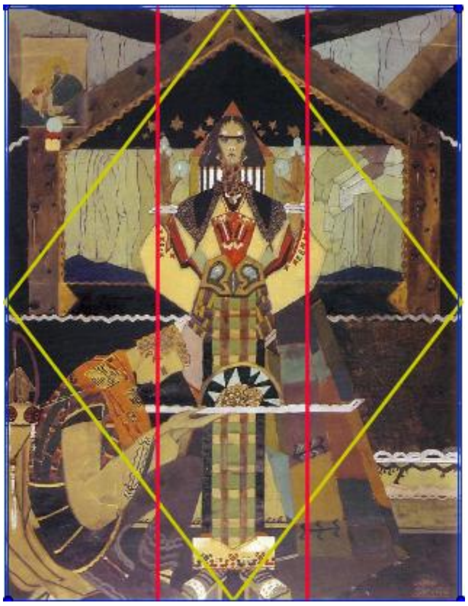


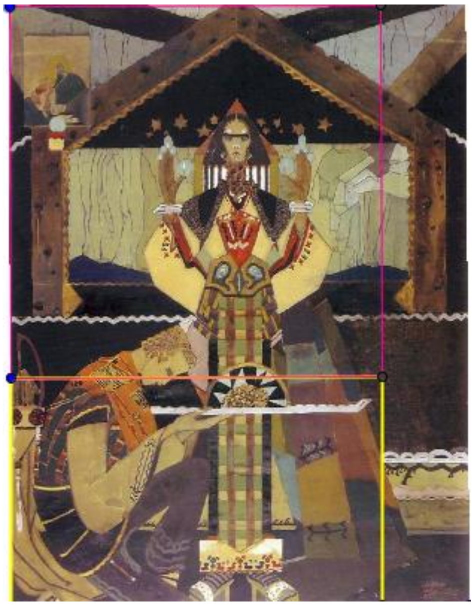












Materials used

Chehlarova, **T., E. Sendova.** Stimulating different intelligences in a congruence context. In: Constructionist approaches to creative learning, thinking and education: Lessons for the 21st century. Proceedings for Constructionism 2010. The 12th EuroLogo conference. 16-20 August, Paris, France. 2010. ISBN 978-80-89186-65-5 (Proc) ISBN 978-80-89186-66-2 (CD)

http://dmentrard.free.fr/GEOGEBRA/art/ARTGEOGEBRA.htm

http://jmora7.com/Arte/arte.htm

http://www.dossantosdossantos.com/Arte/Arte_com_c%C3%B3nicas.html

Recommended reading

Stephen Skinner, *Sacred Geometry*, Sterling, New York/London, 2009

Priya Hemneway, *Divine Proportion*, Sterling Publishing, New York, 2005

Matila Ghyka, *The geometry of art and life*. New Dover publications, Ins, York

Mario Livio, *The Golden Ratio*, Broadway Books, New York, 2002

Scott Olsedn, *The Golden Section*, Walker&Company, New York, 2006