GeoCaching – how to find it ... using satellites

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1 Introduction

A lot of people use GPS devices to find out their own position, or to find a route between two points. More and more cars are equipped with them, and sometimes it seems that the good old map-reading is on the way out. But you can do something else with your GPS device. Not only does it allow you to find your way to a certain city or the next gas station, it also allows you to find so-called geocaches. Now, what is a geocache? A geocache is a container (including a logbook) of some sort (sizes range from very small ones the size of a screw head to very big ones the size of a bucket) that somebody hid, then published its GPS coordinates on a webpage. Your job is to use these coordinates and your GPS device to find the container, then log this find both physically in the logbook and virtually on the webpage. Sounds easy, right? Well, often it is not as easy as you might think. Here you can find out how it works, what you need, and what is has to do with mathematics.

2 GPS explained

GPS (Global Positioning System) is based on three-dimensional geometry, using a number of satellites that are in earth orbit. These satellites are continually sending signals that can be received and interpreted by a GPS receiver (sometimes also called a GPS device, or simply – but less accurately – a GPS). The signals contain the satellites position, the time when the signal was sent, and additional information about satellite health and the other satellites. With the help of the signals of (usually) at least four satellites, it is possible to calculate the position of the GPS receiver. Not counting error corrections (which make for a fairly complicated calculation), the calculation of position works like this: The receiver computes the difference between the time t_s the signal was sent by the satellite and the time t_r the signal was received by the device. As the signal travels with the speed of light c, this allows the computation of the distance d to the satellite as $d = c \cdot (t_r - t_s)$. Now we know that we are (or actually the GPS device is) at a distance of d to the first satellite. We also know this satellites position, so we only need to think "what is the set of points that have a given distance to a fixed point?" In the plane, the answer would be "a circle", but as we are in (three-dimensional) space, the answer is "a sphere" (to be more exact: "the surface of a sphere"). So, with only one satellite signal, we would only know that we are somewhere on the surface of this (virtual) sphere. With the signal of a second satellite, we can construct a second sphere, and now we know that we are on the surface of both of these spheres, i.e. on the intersection of two sphere surfaces. The intersection of two sphere surfaces is a circle. A third satellite provides us with yet another sphere, and the intersection of these three surfaces is a set of two points. If these two points are far away from each other, this might be enough, as we usually have other information to decide which one of these two points is our actual position (e.g. usually you would know whether you are somewhere in Austria or somewhere near the South Pole). But then, the two points may be fairly close together, and it would be nice to know whether you are near your hotel or have another 20 km to hike. So for all practical purposes we need the signal from a forth satellite, which results in exactly one point (again, we ignore possible errors and their correction and remain within the description of an ideal case).

2.1 Excursion: Intersecting three circles

To help understanding the principles of GPS, we will consider a (hypothetical) two-dimensional case and intersect circles instead of spheres in the Dynamic Geometry System GeoGebra (the file can be found here). This case is not so hypothetical after all, it has been used for navigation on the earths' surface (for small distances, it is almost a plane surface), where fixed transmitters instead of satellites were used in a process called trilateration (in historic times, several different methods have been used to navigate. If you are interested, you can find more about it in [1]). If we start with one signal (let's say the transmitter is located in Passau), we would receive a circle:

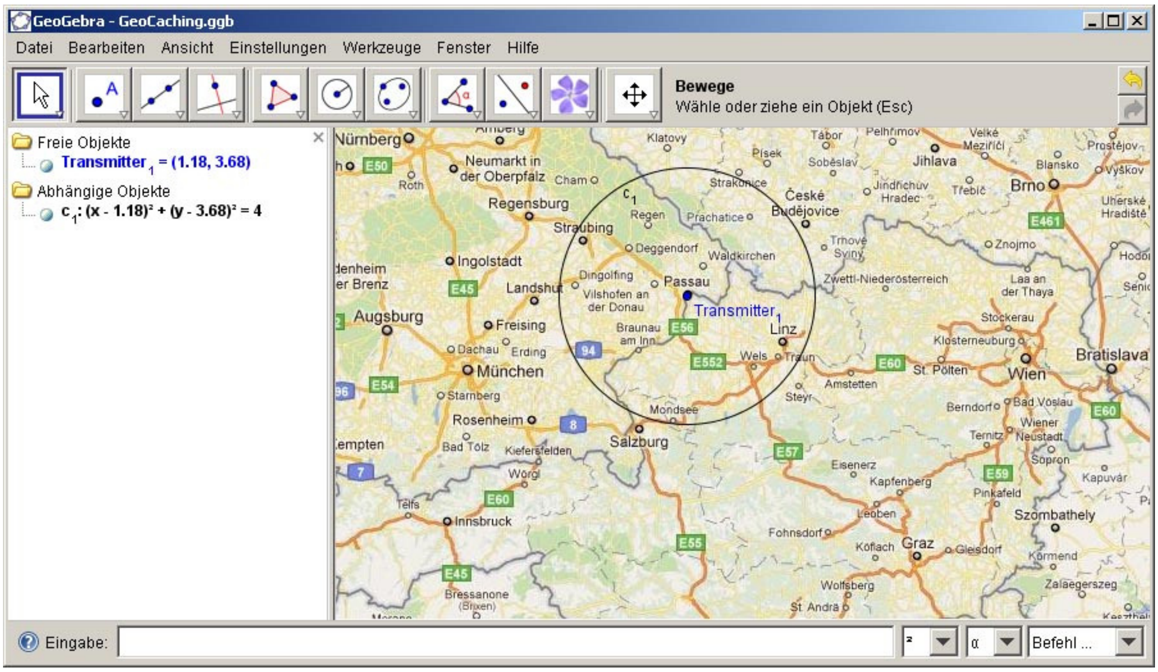


Fig.1 With one transmitter, we know that we are somewhere on this circle

Using a second transmitter (let's say it is based in St. Pölten), we would get this:

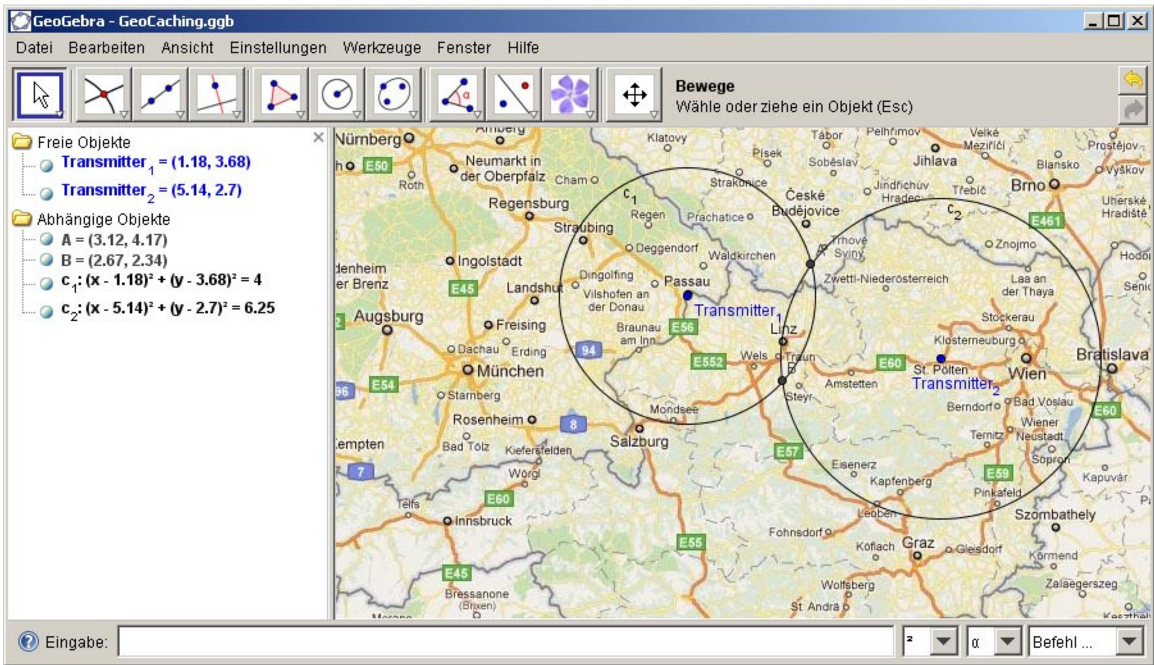


Fig.2 With two transmitters, only two possible positions remain

We now know that we are either near the Austrian town of Steyr or near the Czech-Austrian border. To get our exact location, we need a third transmitter (which let's say is based in České Budějovice):

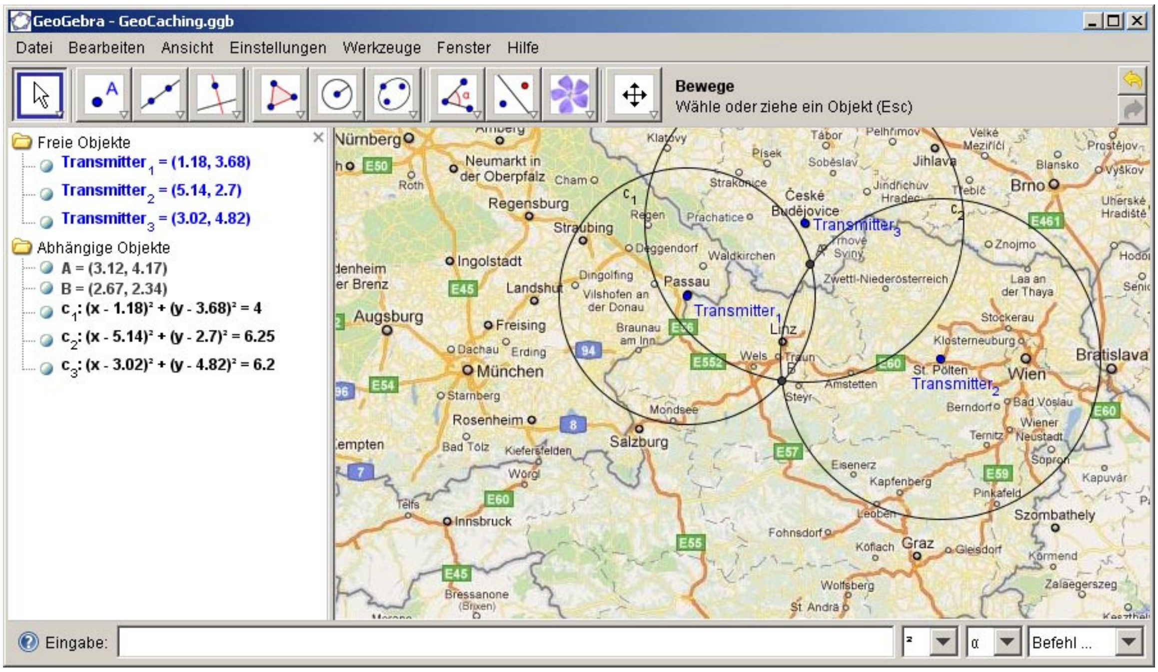


Fig.3 Three transmitters pinpoint your location – ideally

In this case, we would now know that we are indeed at a position near the Austrian town of Steyr. Now this is of course an idealized case: Errors in time measurement etc. would in reality lead to a situation like this:

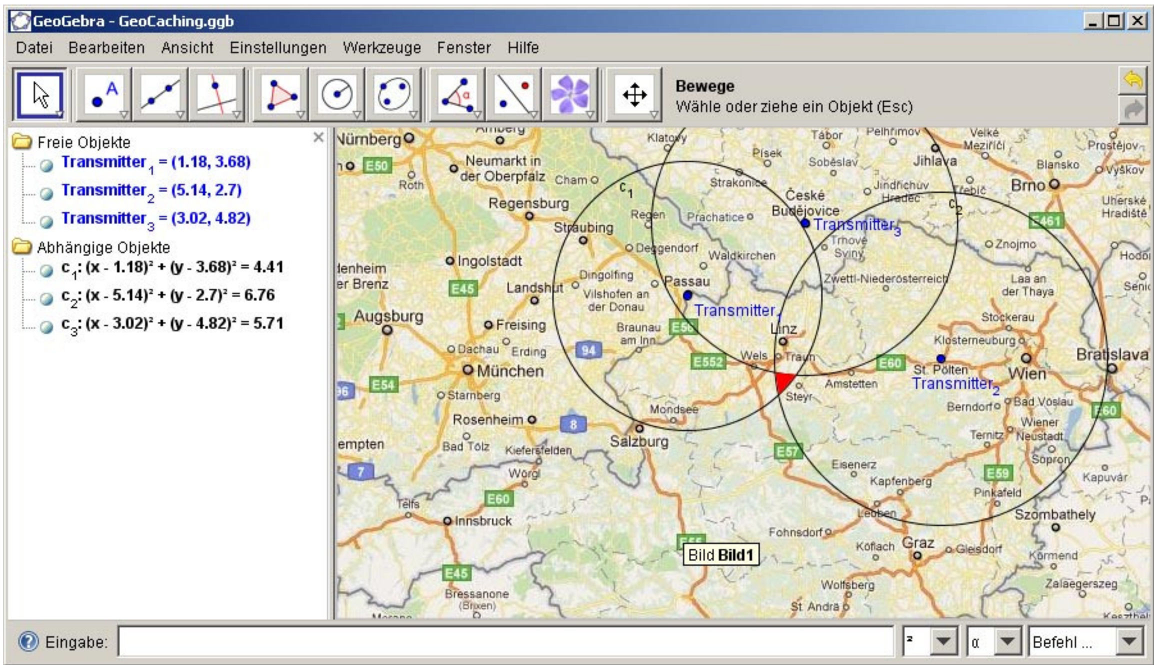


Fig.4 Errors make your position not quite as reliable as it seems

So in reality we are not able to pinpoint our exact location, but we only get an approximate position. More transmitters (i.e. more satellite signals) would reduce the margin of error, but some inaccuracy always remains! Depending on local conditions, weather conditions, the number of satellites that are in your line of sight etc., the accuracy of GPS can vary a lot. Under ideal conditions (open field, clear view of the sky), you can locate your position with an accuracy of about 5 m. If you are in the forest or between high buildings, this may deviate to 20 m or more (see e.g. this data of a jogging trip in Vienna).

Tasks:

[1] Use a Computer Algebra System to calculate the intersection of two and three circles.

[2] Use either the results of Task [1] or the GeoGebra file to answer the following question: If each of the three circles has a radius of 200 km, and the radius can deviate by 0.01%, how many meters can the resulting position deviate?

3 Let's go find something!

3.1 Coordinate systems

Our original goal was to find something that someone else has hidden and has then published the coordinates of the hiding place on a webpage. If we speak about coordinates, we have to first agree which coordinate system we use. In school, we are used to using mainly Cartesian coordinates. These are fine on a plane surface, but not very useful on the surface of the earth, which is essentially the surface of a sphere (counting peas: The earth is actually what is called a geoid, which is kind of a "flattened" sphere). In this case, we have to use a spherical coordinate system (basically the three-dimensional equivalent of the well-known polar coordinate system). Each spherical coordinate system requires a fixed origin (from where the distance is measured) and two fixed planes (from where the angles of longitude and latitude are measured). In GPS navigation, people usually use the WGS84 system. Its fixed origin is the centre of the Earth; the fixed planes are the plane through the equator and a plane through the "zero meridian" near Greenwich, UK. Each point therein can be described by two (if we just want to know the position on the earths' surface) or three (if we also want to know positions under water or in the air) coordinates: A latitude, a longitude, and (eventually) an altitude (usually, the altitude is used instead of the distance to the earth centre). Both the latitude and the longitude are measurements of angle, which are usually given in degrees and (angular) minutes. The latitude is counted north or south of the equator, the longitude west or east of the zero meridian. A typical WGS84 position would look like this: N 48° 12.507, E 016° 22.331.

Tasks:

[3] Use Google Maps or Google Earth or a similar tool to find out where the position with the above coordinates actually is.

[4] Find out the WGS84 coordinates of your home and of your school.

[5] How many meters does your position change, if you vary the longitude by 1°?

3.2 Interpreting GPS data

If you set your GPS receiver to record the track data of your search for a geocache, you usually obtain a long list of numbers. To interpret these numbers, it is useful to use spreadsheet software, e.g. Excel. In the aviation material, you can find out how to prepare the GPS data and also how to prepare graphical representations of it. An example for typical GPS data from a geocaching trip can be found here (Excel table with data) and here (original GPS data).

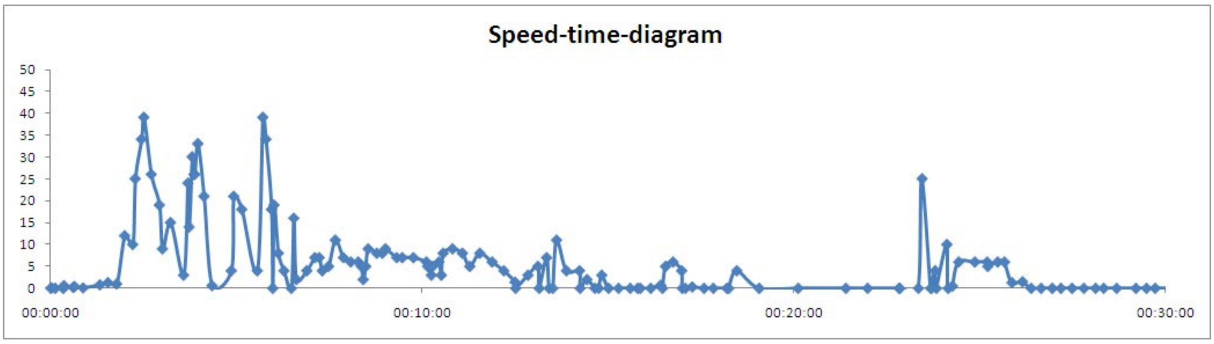
Tasks: (Use the given data or your own GPS data for the following tasks)

[6] How much time did the geocaching trip take?

[7] What distance has been covered?

[8] What has the average speed been?

Using the same methods as in the aviation material, you can construct a speed-time-diagram and a speed-distance-diagram:



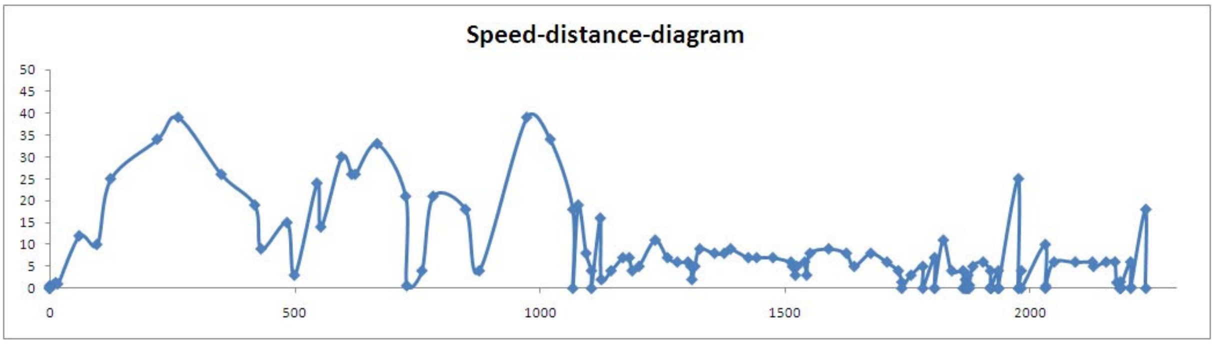


Fig.6 Speed diagrams of a geocaching trip

On a map, the trip looks like this:

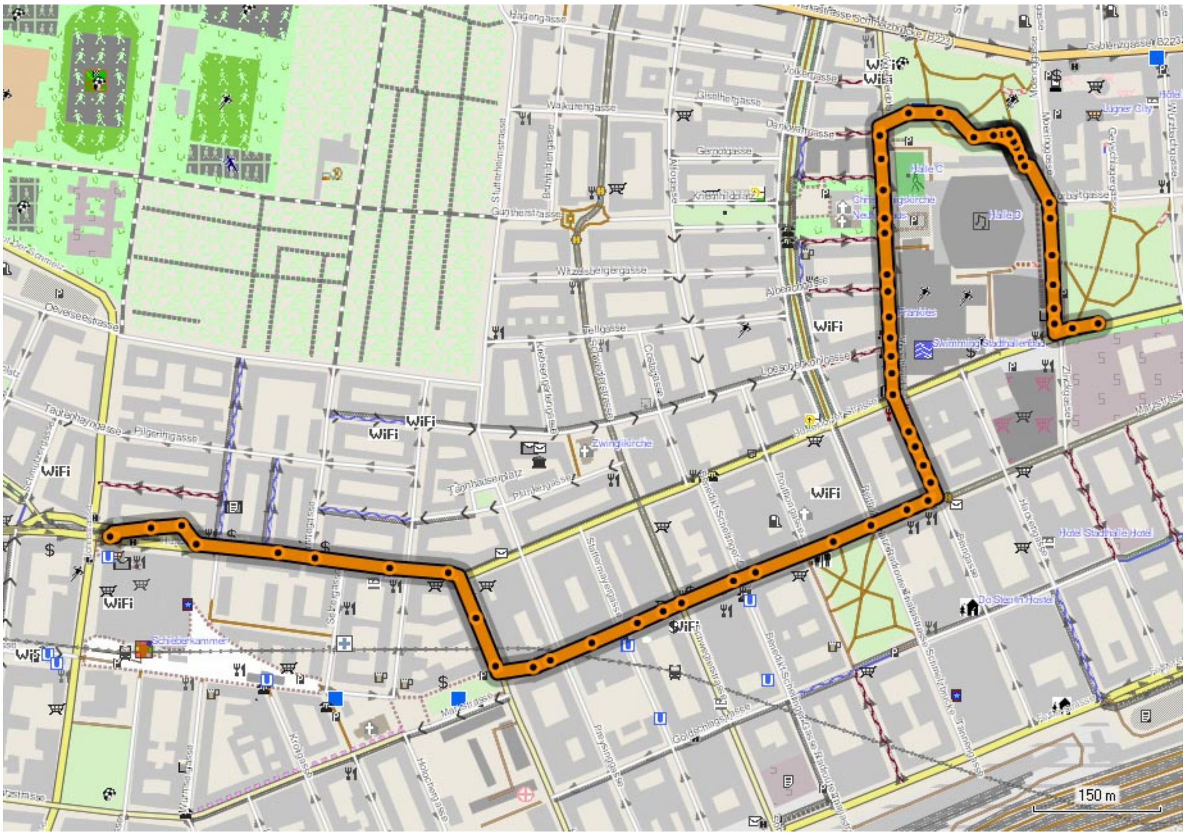


Fig.7 Map of a geocaching trip

Tasks: (Use the given data or your own GPS data for the following tasks)

[9] What distance was covered by public transportation, what distance was covered by foot?

[10] Where on the map do you think the geocache was (hint: it is not at the beginning or the end of the orange line)?

[11] If you would have used a bike with an average speed of 20 km/h, would you have been faster? Keep in mind that you cannot go everywhere with a bike. Fit the data into Google Earth to find out where a bike is usable and where not.

It is easy to think of many more questions that may be answered using GPS data, see e.g. the materials here. In any case, you can see that maths is indeed not only for school, but for real life!

References

[1] Taylor, E. G. R. *The haven-finding art; A History of Navigation from Odysseus to Captain Cook*, American Elsevier Publishing Company, New York, 1971

[2] http://www.gps.gov/systems/gps/ (October 14, 2011)

[3] WGS84 implementation manual (October 14, 2011)

[4] http://www.geocaching.com (October 14, 2011)