Modelling optical lenses with Dynamic Geometry Software

Andreas Ulovec

1 Introduction

In optics when it comes down to show the path of rays of light through glass, lenses or systems of lenses, many physics teachers groan – the experiments are quite complex, and you need a lot of equipment. It is difficult enough to show a ray of light in air – you need smoke, dust or any other way of making light visible. To show the path of light in materials, you need special equipment – smoke glass lenses etc. Now that's not always available, and adjustments to the system can usually only be done by removing one piece and putting another piece in. To see what happens if you make a lens thicker, you have to take out the current lens and put in the new one. Students can then observe the situation before the change and after the change – but it is not exactly a gradual change that lets them observe how the path of light actually changes. We want to demonstrate how you can show the path of light through a lens with the help of dynamic geometry software (DGS).

This material can be useful for science teachers, who can use it to model experiments with lenses, reflection and refraction – not *instead* of the actual experiment (if one sees experiments only in simulation, the pedagogic value is not quite the same), but *complementing* it. It can as well be useful for mathematics teachers. Well, now where is the mathematics? There is a lot of it in there! If a ray of light hits the glass surface of an optical lens, a part of it gets reflected back in a certain angle, and another part penetrates the glass and continues there, in another angle. The same happens when the light reaches the other surface of the lens – again mathematics is required to calculate the angle in which the light is reflected and refracted. For ideal lenses, there is an easy equation calculating these effects – but this is just a model, and it does work well only with thin lenses and with light falling in near the centre of the lens. With thicker lenses and light being more off-centre, the calculations become more complex, and from the equations alone it would be difficult to see what happens. With DGS it is possible to simulate the properties of a lens without actually having to use a lens, laser light, etc. But even for the DGS, we need mathematics to create the simulation in the first place.

2 Easy beginnings – light hits a plane surface

2.1 Reflection

When a ray of light hits a plane glass surface, a part of it is reflected. The *law of reflection* says that the angle of incidence (between the ray of light and the *normal*) is equal to the angle of reflection:

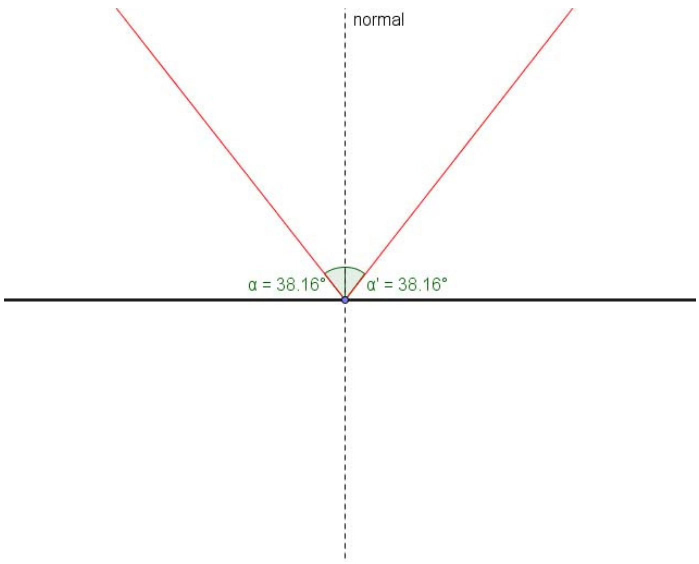


Fig.1 Reflection of light at a plane surface.

2.2 Refraction

Not all the light is reflected, though (only in case of an ideal mirror or in case of *total reflection*). Part of the light is *refracted*, i.e. is passing into the glass. Due to the different light speeds in different media (here: air and glass), the light is not just passing straight through the medium, but is refracted, i.e. passes through the medium in a different angle than the angle of incidence. This angle is calculated by *Snell's law of refraction*, which states that the angle of refraction α'' is related to the angle of incidence α by:

$$\frac{\sin \alpha}{\sin \alpha''} = \frac{n_2}{n_1}$$

Where n_1 is the refractive index of the first medium (here: air, which has a refractive index of approximately 1), and n_2 is the refractive index of the second medium (here: glass, which has a refractive index that depends on the sort of glass that is used; a typical value would be 1.5). This results in the following:

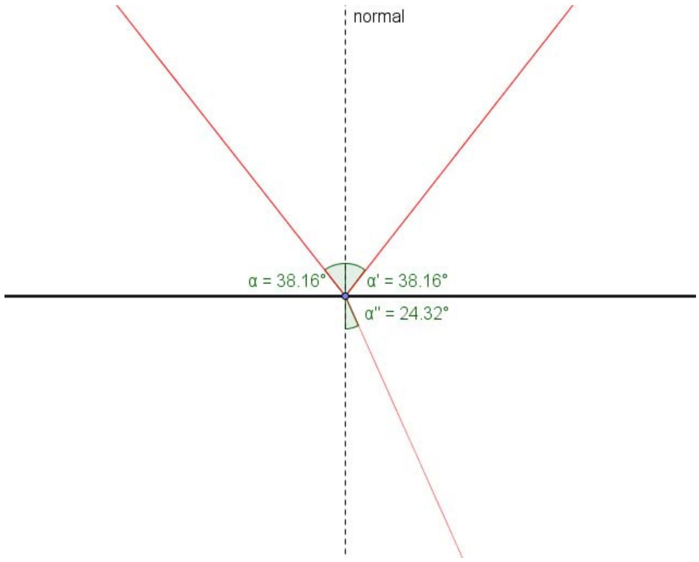


Fig.2 Reflection and refraction of light at a plane surface.

Tasks:

[1] Change the angle of the incoming ray to 45°. Calculate the angle of refraction with a refractive index of 1.5. How much would this angle change, if we would use a high-refractive glass with a refractive index of 1.9?

[2] What would be the angle of the incoming ray of light, if the angle of refraction is 45°?

3 What now? – Light hits a spherical surface

In the case of a plane surface, we measured the angles of incidence and reflection with respect to the *normal*, i.e. the line that is normal (meaning in a right angle) to the given plane surface. If the surface is spherical, we have to generalise the term normal: It means the line that is normal not to the whole surface (which would not make sense in a case other than a plane), but to the tangent (to be exact: to the tangent plane) in the point where the line meets the surface:

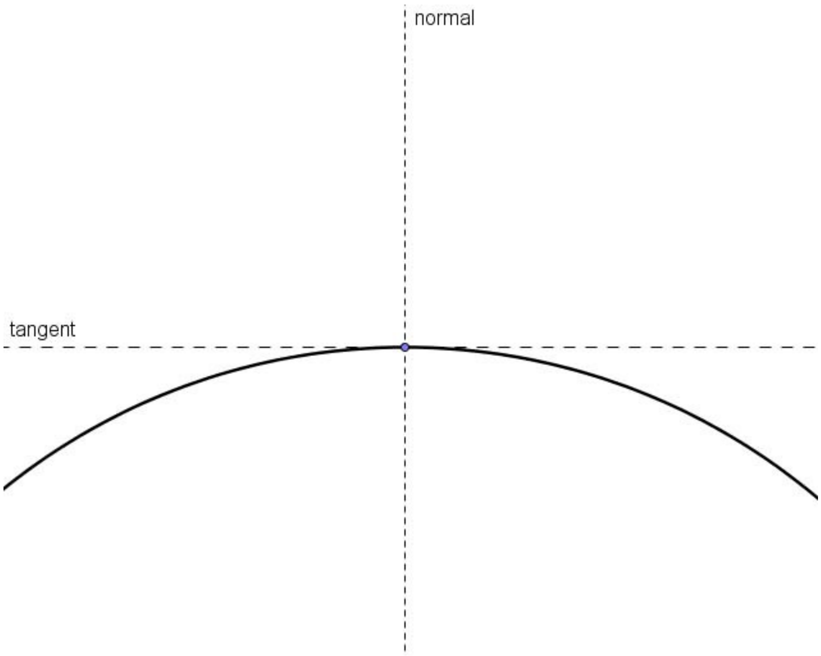


Fig.3 Normal on a spherical surface.

With this generalised definition of a normal, we can calculate reflection and refraction the same way as we did in the plane case:

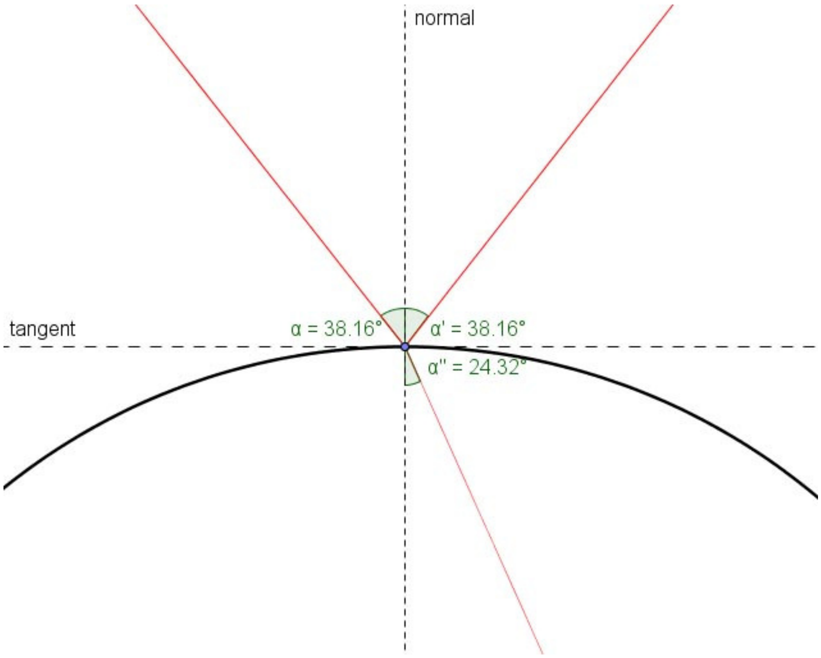


Fig.4 Reflection and refraction of light at a spherical surface.

Now this looks exactly the same as in the plane case. The difference becomes apparent when we shift the ray of light. In the plane case, the angle of refraction does not change, while in the spherical case it does:

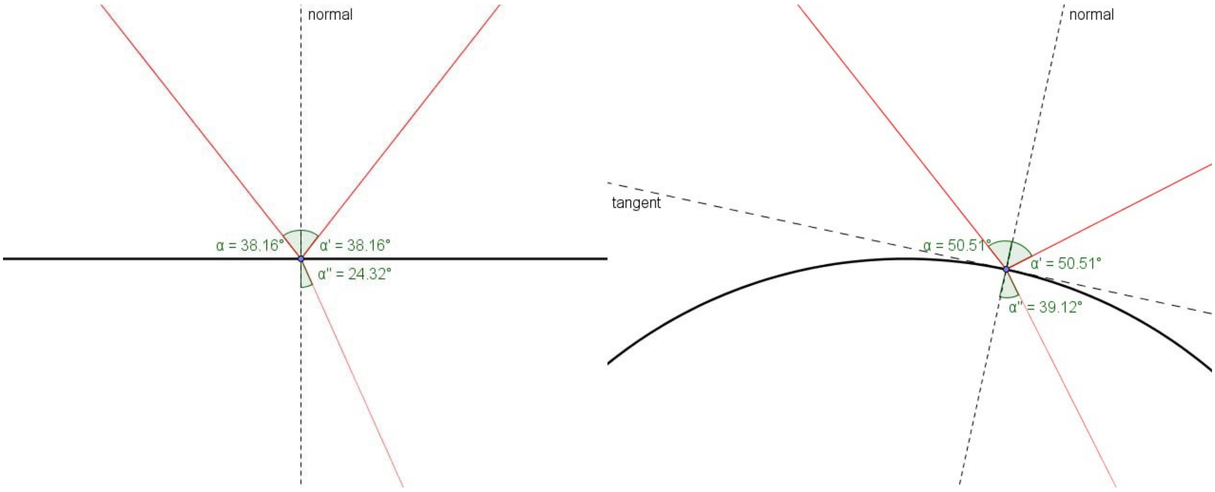


Fig.5 Reflection and refraction of the same ray of light at a plane surface vs. a spherical surface.

This means that for a spherical surface, shifting the ray of light (or the surface) affects the way in which the light is reflected and refracted.

3.1 Concave, convex, confusing?

A spherical surface (as viewed from one side) can have two principal forms – it can be concave, i.e. the inner part curves away from the observer as compared to the outer part, or it can be convex, i.e. the inner part curves towards the observer as compared to the inner part. Confused? It really is easier to see in a figure:

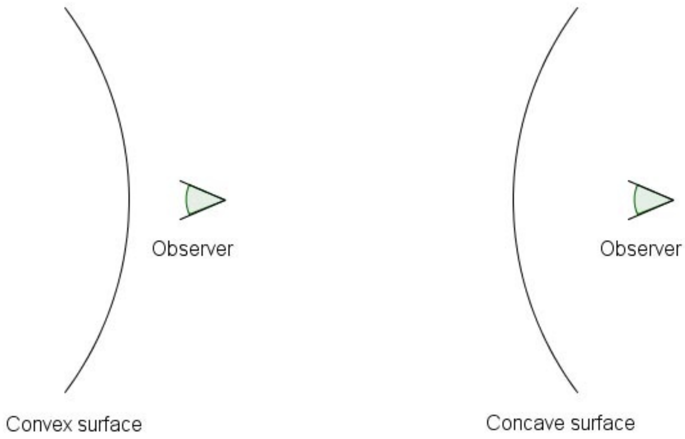


Fig.6 Convex and concave surfaces

In both the convex and the concave case, we construct the tangent and the normal the same way.

4 Doing this twice makes a lens …

In an optical lens, we have this process twice – once when the light enters the lens, and again when the light leaves the lens.

We basically have to do the same construction as before, only with the roles of the refractive indices reversed, i.e. the first medium is glass, and the second medium air. Combining these two, we get the model of a spherical lens:

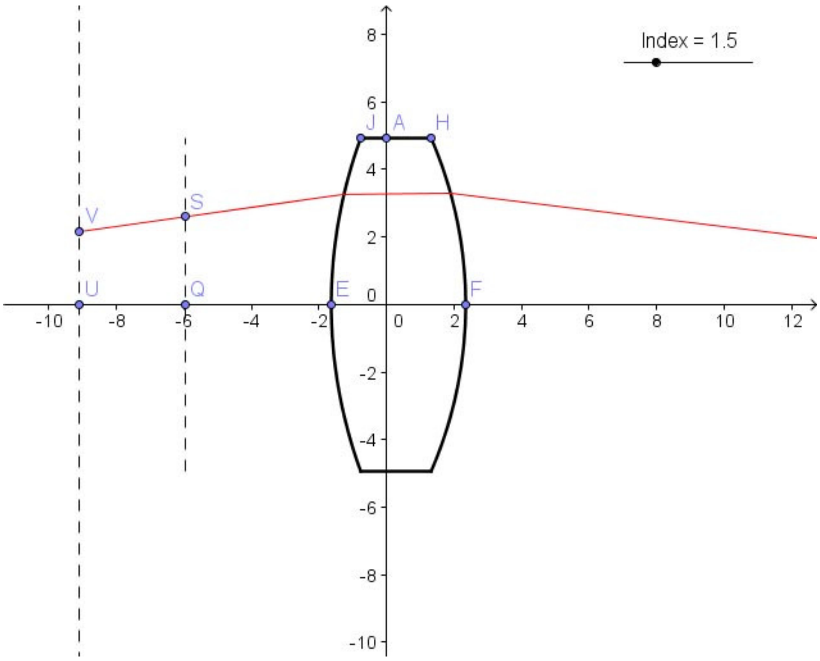


Fig.7 GeoGebra model of an optical lens

With this model, it is fairly easy to answer questions about the properties of the lens without having to manipulate with a real glass lens.

Tasks:

[3] If the light is coming in parallel to the horizontal axis, the outgoing ray of light intersects the horizontal axis at the so-called *focal point* of the lens. If the lens is symmetrical, has a diameter of 10 cm, its thickness in the middle is 4 cm, its thickness at the border is 1.5 cm, where is its focal point?

[4] Does the intersection point change if you move the incoming ray toward the border of the lens?

[5] If the lens is not symmetrical (i.e. if the curvature of the two surfaces is not equal), does the focal point change if you retain the same thickness of the lens, or does it remain the same? Try it out and try to come up with an explanation!

5 This lens is not ideal – but close enough!

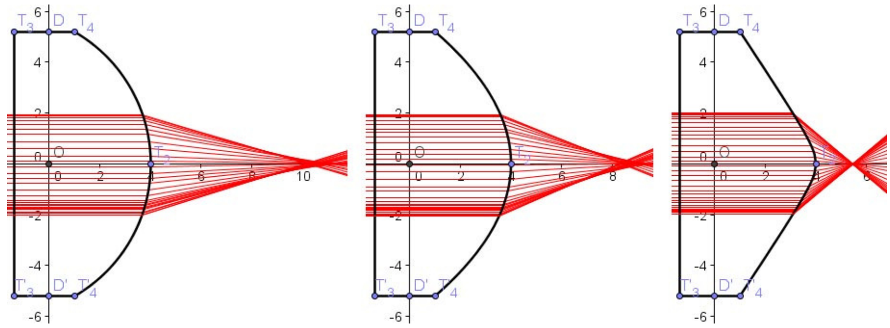
As we have seen above, if we move an incoming parallel ray of light away from the centre of the lens towards the outer regions, the intersection point of the outgoing ray of light with the axis does change – the farther away we are from the centre of the lens, the more the intersection point deviates from the (intended) focal point of the lens. The reason for this is that we have a *spherical lens*, i.e. a lens whose surfaces are sections of spheres. The farther we move away from the centre of such a lens, the worse the optical properties of it become. Depending on the actual purpose of the lens, we may also use a *parabolic lens*, i.e. a lens whose surfaces are paraboloids, or a *hyperbolic lens*, i.e. a lens whose surfaces are hyperboloids. In such a lens, the above-mentioned intersection point would be always in the focal point of the lens, regardless of the distance of the incoming ray from the centre.

Tasks:

[6] Calculate the equation for a symmetric parabola passing through the points F and H. Move the points F and H, and find out when the parabola deviates visibly from the spherical surface.

We can clearly see that only for very thick or very large lenses there is a visible difference between spherical and parabolic surfaces. For a lot of practical purposes, the difference is negligible or at least acceptable, and mostly spherical lenses are used, mainly because it is fairly easy and inexpensive to mechanically create a spherical surface, as opposed to a parabolic or a hyperbolic surface, with certain exactness.

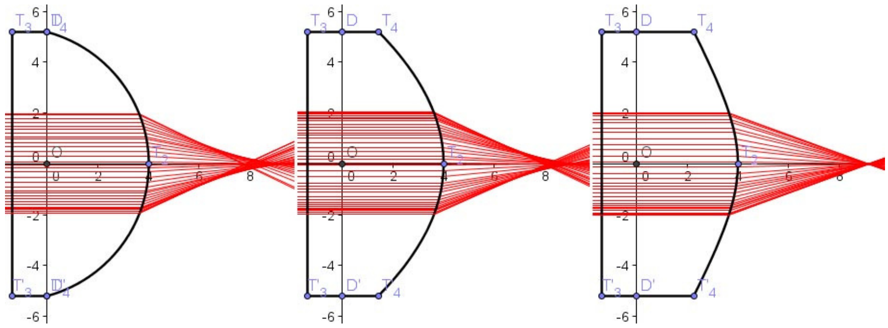
Here is a comparison between a spherical lens, a parabolic lens, and a hyperbolic lens with equal thickness and equal index of refraction:



spherical lens parabolic lens hyperbolic lens

Fig.8 Comparison of the 3 basic lenses with same dimensions

As we can see here, the focal point is different, even when the dimensions are the same. To give a better comparison, we show three lenses with the same focal point and same central thickness:



spherical lens parabolic lens hyperbolic lens

Fig.9 Comparison of the 3 basic lenses with same focal point

We can clearly see here that in the spherical lens, the rays deviate fairly quickly from the focal point, while it is better with a parabolic lens, and best with a hyperbolic lens. Yet for thin lenses (meaning the thickness is much smaller than the diameter of the lens), e.g. for most glasses, cameras etc., the spherical lens works very well:

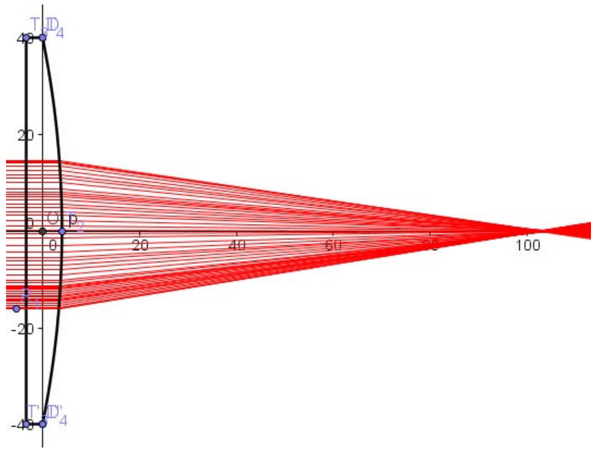


Fig.10 Thin spherical lens

Tasks:

[7] Look for objects in your environment. Where can you find a lens (or more than one)? Try to find out whether it is a spherical, parabolic, or hyperbolic lens, or another form.

References

[1] http://www.geogebra.org (October 14, 2011)